Structured matrix methods for polynomial computations

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Many polynomial computations can be expressed in matrix form, which allows the extensive knowledge of matrix analysis and computational linear algebra to be employed. The matrices that arise in univariate polynomial computations are, in many applications, Tœplitz, or formed by the concatenation of two Tœplitz matrices, for example, the Sylvester resultant matrix. These are examples of structured matrices, and this talk will consider some polynomial computations for which structured matrices are required.

The main difficulty that arises in these computations is that many of them are ill-posed because a continuous random change in the coefficients of the polynomials may cause a discontinuous change in the output. Two examples of this are polynomial division and the computation of the degree of the greatest common divisor (GCD) of two polynomials. In particular, even if $\hat{g}(y)$ is an exact divisor of $\hat{f}(y)$, the division of their perturbed forms, $(\hat{f}(y)+\delta\hat{f}(y))/(\hat{g}(y)+\delta\hat{g}(y))$, is, with high probability, a rational function. This is an unsatisfactory result because it is dominated by the perturbations $\delta\hat{f}(y)$ and $\delta\hat{g}(y)$, and it does not consider the proximity of the given inexact polynomials $\hat{f}(y)+\delta\hat{f}(y)$ and $\hat{g}(y)+\delta\hat{g}(y)$ to their exact forms $\hat{f}(y)$ and $\hat{g}(y)$, respectively. A very similar situation arises with regard to the computation of the GCD of two polynomials.

This talk will show that structured matrix methods allow excellent results to be obtained for some important operations of polynomials. These operations include:

- The computation of the degree of an approximate greatest common divisor of two polynomials whose coefficients are corrupted by additive noise.
- The calculation of a structured low rank approximation of the Sylvester resultant matrix.
- The approximate factorisation of two noisy polynomials whose theoretically exact forms have a non-constant common divisor.
- The computation of the division $h(y) = f(y)/g(y)$ of the noisy polynomials $f(y)$ and $g(y)$, where the division of their exact forms $\hat{h}(y) = \hat{f}(y)/\hat{g}(y)$ is a polynomial. This problem reduces, therefore, to performing the division $h(y) = f(y)/g(y)$ such that $h(y)$ is a polynomial and not a rational function.
- The computation of multiple roots of a polynomial, when the coefficients of the given polynomial are corrupted by additive noise, such that its roots are, with high probability, simple. In this circumstance, the algorithm 'sews' together the computed simple roots that originate from the same multiple root.

Many examples, using high degree polynomial whose theoretically exact forms have many multiple roots and whose coefficients are corrupted by additive noise, will be shown in order to demonstrate the class of problems that structured matrix methods can solve.

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