Distributed Adaptive Consensus Protocol with Decaying Gains on Directed Graphs ⋆

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Abstract: In this paper we present a distributed adaptive consensus protocol, that solves the cooperative regulator problem for multi-agent systems with general linear time-invariant dynamics and directed, strongly connected communication graphs. The protocol addresses the problems of recent distributed adaptive consensus protocols with large or unbounded coupling gains. These problems are solved by introducing a novel coupling gain dynamics that allows the coupling gains to synchronize and decay to some estimated value. Unlike the static consensus protocols, which require the knowledge of the smallest real part of the non-zero Laplacian eigenvalues to design the coupling gain, the proposed adaptive consensus protocol does not require any centralized information. It can be therefore implemented on agents in a fully distributed fashion.

Keywords: cooperative control, adaptive control, multi-agent systems.

1. INTRODUCTION

In past few decades an increasing demand for the cooperation of multiple interconnected systems initiated a great progress made in the design of distributed controllers for networked multi-agent systems. The inspiration came from the collective animal behaviour such as schooling of fish, flocking of birds, herding of quadrupeds and swarming of insect.

The designs of distributed controllers were motivated by the previously developed theoretical results in the centralized control. When the centralized controller is used for the control of a network of agents, the controller views it as a single complex system, therefore the complexity of the centralized controller increases with the complexity of the network. In most applications the centralized controller can not observe the full state information due to communication constrains between agents. Moreover, centralized controller might fail when the network topology changes, e.g. an agent or an edge is added or dropped. Therefore the distributed control approach was developed for the control of the multi-agent systems. It handles all drawbacks of the centralized approach and enjoys many advantages, such as robustness, flexibility and scalability.

The basic distributed consensus protocols for formation control in networked multi-agent systems are introduced by Fax and Murray (2004), Olfati-Saber and Murray (2004), Olfati-Saber et al. (2007) and Ren et al. (2007). Various approaches of design of distributed controllers on directed communication graphs are summarized by Zhang et al. (2012). The passivity based design of cooperative controllers is introduces by Arcak (2007). A unified viewpoint on design of consensus regulator on directed graph topologies using the synchronizing region is introduced by Li et al. (2010). The design of distributed controllers and observers using state or output-feedback in continuous and discrete-time is considered by Zhang et al. (2011), Zhang et al. (2012) and Hengster-Movric and Lewis (2013).

The static consensus protocols, e.g. by Li et al. (2010) and Zhang et al. (2011) are very popular in the community because of their well developed and simple controller design. However, because the centralized information (knowledge of the graph topology) is required by each agent by the design, they are not fully distributed.

The recently developed adaptive consensus protocols by Li et al. (2013) and Li et al. (2015) propose a solution to this problem. Since they do not rely on any centralized information, the distributed controllers of agents can be implemented independently without using any global information. Nevertheless the benefit from distributiveness suffers from the high control effort and weak robustness.

In Knotek et al. (2016) we present an adaptive consensus protocol to solve the cooperative regulator problem on undirected graphs. The protocol introduces a novel coupling gain dynamics that forces the coupling gains to synchronize and decay to some estimated value. This solves the above mentioned problems of recent adaptive consensus protocols. In this paper we expand these results to solve the cooperative regulator problem on directed strongly connected communication graphs.

This paper is structured as follows. Section 2 introduces the basic notation and graph preliminaries used throughout the paper. Section 3 states the problems that are being solved by the novel adaptive consensus protocol presented in Section 4. Numerical simulations of the introduced protocol are given in Section 5. Section 6 concludes the paper.

⋆ This work was supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS16/232/OHK3/3T/13.

2. PRELIMINARIES

In this paper the following notations and definitions are used. $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices. A matrix $M = \mathrm{diag}(v)$ for $v \in \mathbb{R}^n$ denotes $\mathbb{R}^{n\times n}$ diagonal matrix with elements of the vector v on the diagonal. Positive (semi)-definite symmetric matrix is denoted by $M \succ (\succeq) 0$.

A directed graph is given by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of arcs. An arc is an ordered pair of nodes (v_j, v_i), $v_j \neq v_i$, where v_i is a child node and v_j is a parent node, i.e. the information flows from node v_j to node v_i. A directed path of length N from node v_1 to node v_N is an ordered set of distinct nodes $\{v_1, \ldots, v_N\}$ such that $(v_l, v_{l+1}) \in \mathcal{E}$ for all $l \in [1, N-1]$. A directed graph is strongly connected if there exist a directed path from every node to every other node. In the sequel, we assume the graph $\mathcal{G}$ to be directed, strongly connected and simple, i.e. there are no repeated edges or self-loops $(v_i, v_i) \notin \mathcal{E}, \forall i$.

The adjacency matrix $E = [e_{ij}] \in \mathbb{R}^{N\times N}$ associated with the graph $\mathcal{G}$ is defined by $e_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $e_{ij} = 0$. Let the degree matrix $D = [d_{ij}] \in \mathbb{R}^{N\times N}$ be a diagonal matrix given by $d_{ii} = \sum_{j\neq i} e_{ij}$. Then the graph Laplacian matrix is defined by $L = D - E$. Denote $p \in \mathbb{R}^N$ the left eigenvector of L, such that $p^T L = 0$.

3. MOTIVATION

Consider a graph $\mathcal{G}$, that consists of N identical agents with general LTI dynamics

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^m$ is the input, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. The matrix A is not necessarily stable but the pair of matrices (A, B) is assumed to be stabilizable.

The goal is to design a control law to solve the cooperative regulator problem in the sense of $\lim_{t\to\infty} ||x_j - x_i|| = 0$, $\forall i, j = 1, \ldots, N$ without requiring any centralized information. An adaptive control approach proposes a possible solution to this problem.

There has been several proposed distributed adaptive consensus protocols. The adaptive consensus protocol introduced in Li et al. (2013) solves the cooperative regulator problem on undirected connected graphs. The more recent adaptive consensus protocol by Li et al. (2015) solves the cooperative tracking problem on directed graphs containing a spanning tree with the leader as the root node. The distributed adaptive consensus protocols do not require any global information of a communication graph, therefore they are fully distributed. Nevertheless they introduce also several drawbacks.

Since their coupling gain dynamics contains a quadratic term the coupling gain derivative is a monotonically increasing function and the coupling gain values rise as long as there is some error in states between agents. The farther the initial conditions of the agents, the higher the final values of the coupling gains. The coupling gains might therefore reach higher values than it is needed for the network stability. The coupling gains are decoupled, therefore they end up with different final values and the network gets unbalanced, i.e. the agents react differently to the input signal.

Assuming some noise in state measurements, the coupling gains would permanently rise until they reach some physical bound. Therefore the coupling gains could be just statically set to the boundary value and the adaptive consensus protocol would not be necessary.

To solve the cooperative regulator problem on directed strongly connected graphs and address the above mentioned difficulties, we introduce a novel adaptive control protocol, that allows coupling gains to decay and synchronize.

4. ADAPTIVE CONSENSUS PROTOCOL

Let each agent implement a control input in the form

$$u_i = c_i K \sum_{j=1}^{N} e_{ij}(x_j - x_i), \quad i = 1, \ldots, N \tag{2}$$

where $c_i \geq 0$ is the time-varying coupling gain associated with an i-th agent. The i-th agent dynamics is given by

$$\dot{x}_i = Ax_i + c_i BK \sum_{j=1}^{N} e_{ij}(x_j - x_i). \tag{3}$$

Let each agent implement the coupling gain dynamics

$$\dot{c}_i = \sum_{j=1}^{N} e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i) + \sum_{j=1}^{N} e_{ij}(c_j - c_i) - \ell(c_i - \kappa_i) \tag{4}$$

where $\ell > 0$ is a constant, $\kappa_i \geq 0$ is a constant estimated by the i-th agent and $\Gamma \in \mathbb{R}^{n\times n}$ is the adaptation-gain matrix.

The gain matrices K and Γ are designed by the LQR method. Let $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ be positive definite symmetric matrices, then

$$K = R^{-1} B^T P \tag{5}$$

$$\Gamma = PBK \tag{6}$$

where the positive definite symmetric matrix $P \in \mathbb{R}^{n\times n}$ is the unique positive definite solution of the algebraic Riccati equation (ARE)

$$0 = A^T P + PA + Q - PBR^{-1} B^T P. \tag{7}$$

The introduced adaptive consensus protocol (2, 4) is motivated by Li et al. (2013) and Li et al. (2015), however there are several major differences. The coupling gains are associated with each agent as by Li et al. (2015) and not each interconnection as by Li et al. (2013), but the protocol is more similar to Li et al. (2013). This leads to qualitative changes in the network interactions.

The coupling gain dynamics (4) is not a monotonically increasing function as it was in Li et al. (2013) and Li et al. (2015). It consists of three main terms. The first term on the right-hand side $\sum_j e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i)$ is the non-negative quadratic term motivated by the coupling gain dynamics from Li et al. (2013). Its purpose is to push the coupling gains to higher values until the states get synchronized. The second term on the right-hand side

$\sum_j e_{ij}(c_j - c_i)$ synchronizes the coupling gains and thereby solves the above mentioned difficulties with different and to some extent high coupling gains. The third term on the right-hand side $-\ell(c_i - \kappa_i)$ pushes the coupling gains to κ_i and by this solves the problem with generally high gains. The value of κ_i is estimated by an estimation algorithm. The decay rate ℓ determines the strength of the convergence of the coupling gain c_i to the value of κ_i.

The proper estimation of κ_i is required for the exponential stability of the network dynamic (3, 4). Not estimating κ_i and just setting it to zero would mean that the solution of the network dynamics ends up in a bounded set. This worst case behaviour guarantees that the states of agents can not get arbitrarily far apart from each other and provides time for the estimation of κ_i. The value of κ_i is estimated by an estimation algorithm from the network trajectory. This is thoroughly discussed in the next sections.

In the sequel, we derive the network dynamics (3, 4) and investigate its stability. First we define a general network dynamics (8, 9). Then step by step by modifying the general network dynamics (adding new terms to the coupling gain dynamics) we derive the introduced network dynamics (3, 4). In each step we investigate the stability of the obtained network dynamics. These results we use to conclude the stability of the network dynamics (3, 4).

4.1 General network dynamics

Assume the general network dynamics (3, 4) with $\ell = 0$

$$\dot{x_i} = Ax_i + c_iBK\sum_{j=1}^{N} e_{ij}(x_j - x_i) \tag{8}$$

$$\dot{c}_i = \sum_{j=1}^{N} e_{ij}(x_j - x_i)^T\Gamma(x_j - x_i) + \sum_{j=1}^{N} e_{ij}(c_j - c_i) \tag{9}$$

where decay term $-\ell(c_i - \kappa_i)$ that pushes the coupling gain value c_i to the value of κ_i is omitted.

Assume that the general network dynamics starts with different initial states and different initial coupling gains of all agents. Consider agents with unstable but stabilizable dynamics. If the coupling gains are small the general network dynamics might be unstable and the states of agents might diverge. The adaptive term is positive while there is some error in states of agents, therefore it pushes the coupling gains to higher values. At the same time the coupling gain synchronization term pushes the coupling gains towards each other to synchronize. When the coupling gains rise sufficiently high the general network dynamics becomes stable and the states of agents start to synchronize. When the states of agents get synchronized the adaptive term is zero. If the coupling gains are not synchronized yet the synchronization term synchronizes the coupling gains and they get steady at some common final value. Note that since the coupling gains are non-negative real numbers the coupling gain synchronization term can not push them to negative values.

Starting with agents having a stable dynamics, the general network dynamics behaves the same as in the previous case except that it is always stable because it is stable for any coupling gain values. Note that the control law just speeds up the synchronization of the states of agents. Since all agents end up in one common equilibrium the control law does not need to be implemented to reach consensus.

From the previous analysis, one expects that the states of agents synchronize and the values of the coupling gains rise and synchronize to some finite value for all initial conditions, i.e. the general network dynamics is globally asymptotically stable. This result is confirmed by the simulations.

Note that by the asymptotic stability of a network we mean the stability with respect to the collective dynamics of agents in the sense of $||x_j - x_i|| \to 0$ as $t \to \infty$, $\forall i, j$. Same holds for the exponential stability with addition, that convergence $||x_j - x_i|| \to 0$ as $t \to \infty$, $\forall i, j$ is faster than some exponential function.

The work on the proof of the global asymptotic stability of the general network dynamics (8, 9) is currently in progress therefore we introduce just an idea of the proof. Define the virtual leader $x_0 = \sum_i p_i x_i$ and the virtual tracking error $\delta_i = x_i - x_0$. Assume the coupling gain dynamics (4) with $\ell = 0$, then the network error dynamic is

$$\dot{\delta_i} = A\delta_i + c_iBK\sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i) - \sum_{k=1}^{N} p_k c_k BK \sum_{j=1}^{N} e_{kj}(\delta_j - \delta_k) \tag{10}$$

$$\dot{c}_i = \sum_{j=1}^{N} e_{ij}(c_j - c_i) + \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i)^T\Gamma(\delta_j - \delta_i). \tag{11}$$

Consider a Lyapunov function candidate

$$V = \sum_{i=1}^{N} p_i\delta_i^T P\delta_i + \sum_{i=1}^{N} p_i(c_i - \alpha)^2 \tag{12}$$

where α is some positive constant. The time-derivative of the Lyapunov function candidate along the trajectory of the network error dynamics (10, 11) is

$$\dot{V} = \delta^T[P_1 \otimes (A^TP + PA) - \alpha(P_1L + L^TP_1) \otimes \Gamma]\delta - c^TP_1Lv - \frac{1}{2}c^T(P_1L + L^TP_1)c. \tag{13}$$

where $P_1 = \mathrm{diag}(p)$, $\delta = (\delta_1^T, \ldots, \delta_N^T)^T$ is the vector of virtual tracking errors, $c = (c_1, \ldots, c_N)^T$ is the vector of time-varying coupling gains and $v = (v_1, \ldots, v_N)^T$ is the vector of quadratic forms given by $v_i = \delta_i^T\Gamma\delta_i$. To prove that the general network dynamics (8, 9) is globally exponentially stable it has to be shown, that for every initial condition there exists α such that $\dot{V} \leq 0$.

The general network dynamics (8, 9) synchronizes the coupling gains to one common value that is lower than the final value of the largest coupling gain without using the coupling gain synchronization. This is found to solve the problems of the recent adaptive consensus protocols Li et al. (2013) and Li et al. (2015) with different and partially large gains. Nevertheless the general network dynamics inherits the problem with unbounded coupling gains. Therefore we modify the general network dynamics to allow coupling gains to decay and introduce the uniformly ultimately bounded network dynamics (14, 15).

4.2 Bounded network dynamics

Assume the coupling gain dynamics (4) with $\kappa_i = 0$, $\forall i$, then the network dynamic is given by

$$\dot{x}_i = Ax_i + c_i BK \sum_{j=1}^{N} e_{ij}(x_j - x_i) \tag{14}$$

$$\dot{c}_i = \sum_{j=1}^{N} e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i) + \sum_{j=1}^{N} e_{ij}(c_j - c_i) - \ell c_i. \tag{15}$$

This network dynamics consists of the general network dynamics (8, 9) and an additional decay term $-\ell c_i$ in the coupling gain dynamics that pushes the values of the coupling gains to zero.

Consider agents with unstable but stabilizable dynamics. Assume that the network dynamics (14, 15) starts with different initial states and different initial coupling gains of all agents. Note that the network dynamics might be unstable because of small coupling gains. Then the adaptive term overpowers the decay term therefore the coupling gains start to rise. When the coupling gains rise sufficiently high the network becomes stable and the states of agents synchronize. When the states of agents get close to each other the decay term overpowers the adaptive term and the coupling gains start to decay to zero. Decreasing coupling gains slowly destabilize the network dynamics. When the coupling gains decay sufficiently low the network becomes unstable and the states of agents start to diverge. With the states far from each other the adaptive term again overpowers the decay term and the cycle repeats. Hence one expects oscillatory behaviour of the network trajectory in some set. The term that synchronizes the coupling gains apparently does not have any influence on this oscillatory behaviour. The simulations of the network dynamics (14, 15) from Section 5 confirm the boundedness of its solution.

If the agents are stable then also the network dynamics is stable. The states of agents converge to the equilibrium point and thereby synchronize, therefore the consensus protocol is not necessary. Applying the adaptive consensus protocol one expects faster convergence to consensus. Let the network dynamics start with different initial states and initial coupling gains of agents. The coupling gains start to rise because the adaptive term overpowers the decay term. The states of agents are pushed towards each other to synchronize by the adaptive consensus protocol. When the states of agents get close to each other the decay term overpowers the adaptive term and the coupling gains start to decay to zero. At the same time the states of agents synchronize and the coupling gains tend to zero therefore the states end up synchronized in the equilibrium point and the coupling gains end up being zero.

The work on the proof of the boundedness of the solution of the network dynamics (14, 15) is currently in progress therefore we introduce just the possible sketch of the proof. Define the coupling gain transformation $z_i = c_i - \beta$, where β is some positive constant. Then the network error dynamics (14, 15) in the new coordinates (δ_i, z_i) is

$$\dot{\delta}_i = A\delta_i + z_i BK \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i) + \beta BK \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i)$$

$$- \sum_{k=1}^{N} p_k z_k BK \sum_{j=1}^{N} e_{kj}(\delta_j - \delta_k) \tag{16}$$

$$\dot{z}_i = \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i)^T \Gamma (\delta_j - \delta_i) + \sum_{j=1}^{N} e_{ij}(z_j - z_i) - \ell z_i - \ell \beta. \tag{17}$$

This network dynamics consists of some nominal dynamics and the non-vanishing perturbation term $-\ell\beta$, therefore consider it as the perturbed nominal dynamics. First it has to be shown, that the nominal dynamics is globally exponentially stable and the Lyapunov function and its time derivative are bounded. Then, since the non-vanishing perturbation term is bounded, following (Khalil, 1996, Lem. 5.2) it can be shown, that the perturbed nominal dynamics (16, 17) is uniformly ultimately bounded.

The boundedness of the network dynamics (3, 4) is an important property. It says that for $\kappa_i = 0$, $\forall i$, the solution of the network dynamics is uniformly ultimately bounded, i.e. in the worst case the solution ends up being bounded.

Having uniform ultimate boundedness is necessary, but not sufficient for a practical implementation of the protocol. For that reason we introduce the full form of the proposed coupling gain dynamics (4). In comparison to the coupling gain dynamics (15) it contains an additional term $\ell\kappa_i$, that can cancel effects of the non-vanishing perturbation term $-\ell\beta$ from (17).

Consider the coupling gain dynamics (4), then the network error dynamics reads

$$\dot{\delta}_i = A\delta_i + z_i BK \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i) + \beta BK \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i) \tag{18}$$

$$\dot{z}_i = -\ell z_i + \ell\kappa_i - \ell\beta + \sum_{j=1}^{N} e_{ij}(z_j - z_i) + \sum_{j=1}^{N} e_{ij}(\delta_j - \delta_i)^T \Gamma (\delta_j - \delta_i). \tag{19}$$

Increasing the κ_i, effects of non-vanishing perturbation term are continuously reduced until the condition $\kappa_i = \beta$, $\forall i$ is met. If $\kappa_i \geq \beta$, $\forall i$ the non-vanishing perturbation term is cancelled, the dynamics reduces to the case of nominal dynamics and it gets globally exponentially stable.

4.3 Estimation of κ_i

Now we have to properly estimate the value of κ_i to reach global exponential stability of the network dynamics (3, 4). We require sufficiently high value of κ_i to guarantee global exponential stability but at the same time we want the value as low as possible to minimize the control effort.

To estimate the value of κ_i we propose an algorithm based on the interval halving method. The idea is to increase the value of κ_i as long as the trajectory of the network dynamics oscillates, i.e. it is uniformly ultimately bounded. When the network trajectory oscillates the non-vanishing perturbation term is present in the dynamics. To cancel this term the value of κ_i has to rise until the non-vanishing perturbation term is cancelled and the network dynamics gets globally exponentially stable. The oscillating trajectory implies oscillating coupling gain, therefore the coupling gain c_i is sampled by some sampling frequency f_s and recorded in some time window Δt. The highest and the lowest recorded values are averaged and this average is then used as a new κ_i. This process is repeated all the time. The coupling gain c_i and the trajectory of the network dynamics stop to oscillate when the network dynamics becomes globally exponentially stable. Then also the κ_i reaches the steady state value.

Note that the sampling frequency f_s has to be chosen according to the Nyquist-Shannon sampling theorem. The sampling rate must exceed $2f_{max}$ and the time window $\triangle t$ must be greater than $1/f_{min}$, where f_{max} and f_{min} are the highest and the lowest frequency in the system. The decay-rate and the size of the non-vanishing perturbation are determined by the positive constant ℓ. The decay term can be interpreted as a filtration term therefore the constant ℓ influences also the frequency and the amplitude of oscillations of the coupling gain c_i.

To handle the noise acting on state measurements the value of κ_i is updated only if the difference between the maximal and minimal recorded coupling gain values is greater than some predefined dead-zone. Note that the dead-zone influences just the estimation of κ_i therefore it does not harm the stability of the network. The solution of the network dynamics is uniformly ultimately bounded and with a properly chosen dead-zone corresponding to the measured noise it remains uniformly ultimately bounded however it approaches the exponential stability as close as the dead-zone, and thus the noise, allow.

The simulations verify the theoretical results on the proposed adaptive consensus protocol with the interval-halving estimation algorithm.

5. SIMULATIONS

This section shows the simulations of the proposed adaptive consensus protocol (2, 4) with agents described by linear double integrator dynamics

$$\dot{x_i} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i, \quad x_i = \begin{bmatrix} x_{i_1} \\ x_{i_2} \end{bmatrix}, \quad \forall i. \tag{20}$$

The interval-halving algorithm for the estimation of κ_i is configured to the time window $\triangle t = 5s$ and the sampling frequency $f_s = 10\text{Hz}$. The initial value of κ_i, $\forall i$ is set to zero. Initial conditions of the agents are

$$x_{i_1}(0) \in \langle -10, 10 \rangle, \quad x_{i_2}(0) = 0, \quad c_i(0) = 0, \quad \forall i. \tag{21}$$

The uniform ultimate boundedness of the solution of the network dynamics (3, 4) for $\kappa_i = 0$, $\forall i$ is shown by simulations in Figure 1. The algorithm for estimation of κ_i was not used and κ_i was just statically set to zero. The simulation on a circular communication graph consisting of 50 agents is situated on the top of the figure and the simulation on a communication graph consisting of two interconnected circles each with 25 agents is situated on the bottom. The topology of the communication graph with two interconnected circles is shown in Figure 4.

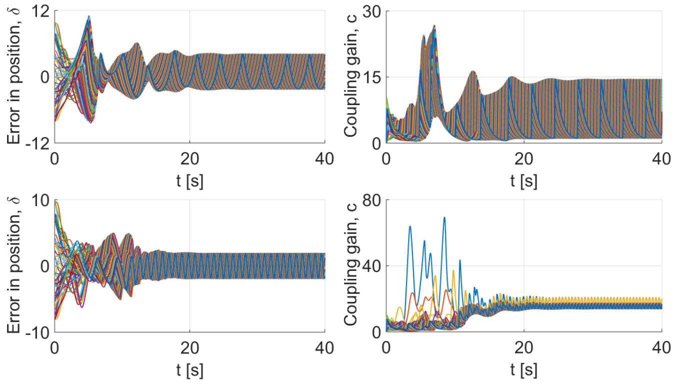

Fig. 1. Simulations of the proposed protocol without estimation of κ_i, for $\kappa_i = 0$, $\forall i$. The simulation of 50 agents on a circular topology is shown on the top and the simulation of two interconnected circles each with 25 agents is shown on the bottom.

In following simulations the final coupling gain value is compared to the minimal coupling gain value required by the static consensus protocol. The static consensus protocol implements the input in the form of (2), but instead of c_i it uses just one coupling gain c for all agents. To guarantee stability by the static consensus protocol the coupling gain value has to satisfy

$$c \geq \frac{1}{2 \min \Re(\lambda_i)} \tag{22}$$

where $\min \Re(\lambda_i)$ is the smallest non-zero real part of the eigenvalues of the Laplacian matrix L.

Simulations of the proposed protocol with the interval-halving estimation algorithm for the estimation of κ_i are shown in Figure 2. The simulation on a circular graph consisting of 50 agents is situated on the top of the figure. In the first few seconds of the simulation the response is uniformly ultimately bounded because of the low coupling gain values. As the coupling gain values rise the network reaches cooperative stability and states synchronize. The coupling gains reach steady state value 61.2 while the calculated lower bound on the coupling gain required by static consensus protocols is 63.4. The simulation on a graph consisting of two interconnected circles each with 25 agents is situated on the bottom of the figure. The topology of the communication graph is shown in Figure 4. The steady state value of the coupling gains reach 15.4 while the calculated lower bound on the coupling gain for the static consensus protocol is 16.

Figure 3 shows the simulations of the proposed protocol with and without noise acting on the state measurements. The simulations have been done on the circular graph consisting of 10 agents. The steady state value of the coupling gains is 3.2 without noise and 5.5 for the case with noise, respectively. The static consensus protocol require the coupling gain value to be greater than 2.6. Since the coupling gain values end in a bounded set and they do not permanently rise as they do in Li et al. (2013) and Li et al. (2015), the proposed protocol with the interval-halving estimation algorithm is found robust to the noise acting on states.

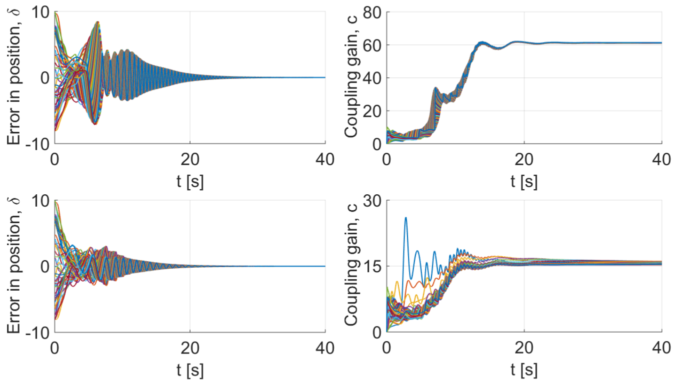

Fig. 2. Simulations of the proposed protocol on a circular graph consisting of 50 agents on the top and on a graph consisting of two interconnected circles each with 25 agents on the bottom, respectively.

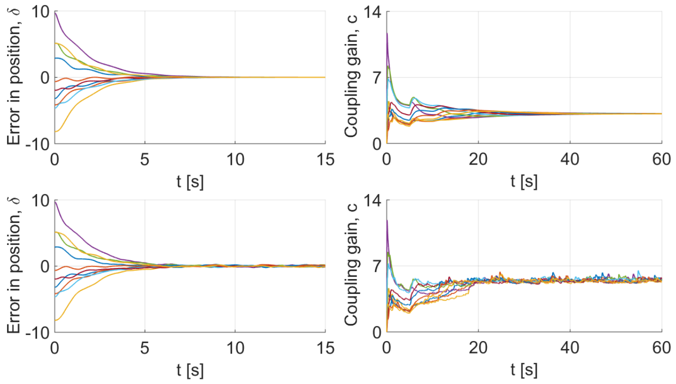

Fig. 3. Comparison of the protocol simulated on a circular topology consisting of 10 agents on the top without and on the bottom with the noise acting on states.

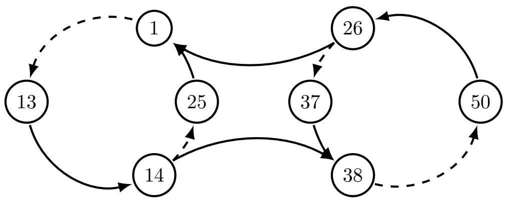

Fig. 4. The communication graph of two interconnected circles each with 25 agents.

6. CONCLUSION

We presented a novel distributed adaptive consensus protocol, that solves the cooperative regulator problem on directed, strongly connected communication graphs. The protocol introduces a novel coupling gain dynamics, that forces the coupling gains to synchronize and converge to some estimated value. This solves the problems of recent adaptive consensus protocols with high gains and consequently large control effort. The value to which the coupling gains converge is estimated by the estimation algorithm based on the interval-having method.

The simulations of the proposed adaptive consensus protocol verify the theoretical results. Due to decay term, the coupling gains in some situations attain lower values than conventional algorithms. The protocol is found robust to the noise in state measurements unlike the recent adaptive protocols whose gains would permanently rise.

The work on the proofs of the global asymptotic stability of the general network dynamics (8, 9) and the uniform ultimate boundedness and the global exponential stability of the network dynamics (14, 15) for different κ_i is still ongoing. The investigation of relation between the decay rate ℓ and the parameters (time window Δt and sampling frequency f_s) is the task of the future research.

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