KTH VETENSKAP OCH KONST

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Review of Mr. Ondřej Šantin's PhD thesis
"Numerical Algorithms of Quadratic Programming for Model Predictive Control"

Model predictive control (MPC) is uniquely positioned to become the dominating advanced control technology: it is inherently multivariable and allows to incorporate a variety of constraints on process variables and control actions. However, it is also computationally intensive since it requires solving an optimization problem when determining the control actions at every sampling instant. This optimization problem has to be solved with extreme reliability, within a predictable (and often short) execution time, and with a small memory footprint. In many application areas, it is our inability to satisfy these computational requirements that hinders the use of model predictive control technology.

In his PhD thesis, Mr. Šantin develops several algorithms for solving the type of quadratic programming problems that appear in MPC applications in a fast, reliable and memory-efficient manner. These algorithms address an important challenge for the scientific community, and may help to enable brand new applications of MPC in industry.

The thesis is very well written and a true pleasure to read. The author treats a single topic (the development of numerical algorithms for solving box-constrained quadratic programming problems that appear in MPC applications) in a condensed, insightful and pedagogical manner. Mr. Šantin demonstrates profound knowledge of the topic and achieves distinct contributions over the state-of-the art.

The author has set the following four main goals for his thesis:

1. To develop a general solver for box-constrained quadratic programming problems suitable for MPC applications on embedded platforms.
2. Ensure that the proposed methods require few iterations, that each iteration is of low computational complexity, and that the convergence properties are insensitive to problem scaling, size and initial iterate.
3. Compare his algorithms with the state-of-the art; and
4. Draw conclusions and directions of future research.

All of these goals have been met. In particular, Mr. Šantin proposes three algorithms called CGNP, PND and NPP, respectively. They all belong to the family of active set methods and use combinations of gradient projections and Newton directions to identify the optimal active set.

The Combined Gradient and Newton Projection method (CGNP) combines a standard gradient projection step with an improvement step in the Newton direction on the face defined by the active constraints. A theoretical analysis of the convergence properties and the computational complexity is presented. Through numerical experiments, it is observed that although the number of iterations of CGNP is higher than competing

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proposals in the literature, it requires a smaller number of total flops for convergence. The Proportioning with Newton Directions (PND) method introduces a "proportionality test" to determine if the active set should be expanded or reduced. This test uses the absolute values of the Lagrange multipliers to decide if the active set should be reduced or expanded, and is shown to be effective in reducing the number of unwanted changes in the active set. Finally, the Newton Projection with Proportioning (NPP) algorithm eliminates two computational bottlenecks of PND by modifying the proportioning step in the PND algorithm. Although the computational effort per iteration increases, numerical experiments indicate that the overall performance of NPP is often superior to PND and CGNP.

All three proposed methods use clever organization of computations to reduce the computational effort and limit the memory requirements. They are all endowed with theoretical convergence guarantees and computational complexity estimates. All three algorithms are very well suited for implementation on embedded devices (as long as the number of states and/or the prediction horizon is not too large). Furthermore, extensive numerical experiments indicate that they are insensitive to problem scaling, problem size and initial iterate.

In a final chapter, the proposed algorithms are compared with the state-of-the art solvers targeting MPC applications. The algorithms significantly outperform the well-known FORCES and FiOrdOs solvers, while the differences with the qpOASES solver are more subtle.

Finally, Mr. Šantin proposes a few selected directions for future research, thus achieving the final of his four goals.

Overall, Mr. Šantin has written a interesting and impressive PhD thesis. It has distinct contributions that advance the state-of-the art in embedded optimization. It is well-written and pedagogical, and describes in an insightful way some of the key tricks and techniques for writing reliable numerical solvers that are able to run on resource-constrained implementation platforms. It also demonstrates the author's profound knowledge of his PhD topic: he knows how to organize computations to limit the computational effort; he is well aware of the computational bottlenecks of his algorithms; and he is able to use the appropriate techniques to balance the per-iteration complexity and its effect on the total number of iterations required to reach a solution. In addition, Mr. Šantin is able to carry out theoretical analyses of his algorithms to guarantee convergence and estimate the computational requirements.

In summary, I consider the thesis a creative and significant contribution in the development of engineering science. The author of the thesis proved to have an ability to perform research to achieve scientific results. I do recommend the thesis for presentation with the aim of receiving the Degree of Ph.D.

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