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Report on the PhD manuscript entitled

The Koopman and moment-sum-of-squares approach for control: computational methods and applications.

by Vit Cibulka

The PhD manuscript proposed by Vit Cibulka addresses some aspects regarding the use of Koopman operators and sum-of-squares (SoS) frameworks to address the modeling and control of nonlinear systems. These two frameworks are tackled separately leading to a manuscript consisting in two almost totally separate parts.

As a whole, the work proposed by Vit Cibulka is of a **high scientific level**. This concerns both the material itself but also the way the content is presented. Despite the possible comments, critics and suggestions that might pop up here and there, the manuscript shows the quality of researcher of Vit Cibulka as well as his remarkable ability to simplify the presentation and choose the level of details to smoothen the readiness of his writing to the reader. His contributions are important within an active and hot research topics in the nonlinear control community.

More precisely, the manuscript is decomposed into Eleven chapters (some of them are one or two pages of conclusion!) that belong to two separate parts respectively entitled: I. KOOPMAN OPERATOR and II. SUM-OF-SQUARES HIERARCHY. Below a brief description of the chapters followed each by some comments and discussion (when appropriate) before a global assessment is given at the end of this report.

Chapter 1 entitled "*Introduction*" gives an overview of the PhD content by introducing the two main topics, namely, the use of Koopman operator and the sum-of-squares frameworks in controlling nonlinear systems through lifting followed by the use of linear problems' tools on the truncated versions of the lifted problems.

Chapter 2 entitled "*The Koopman operator for control*" starts by introducing the Koopman operator in the autonomous and the controlled cases together with their linear truncated versions leading to the convexification of general optimal control problems. The so called Koopman-MPC (KMPC) problem is clearly stated quite elegantly. Then some practical considerations are discussed regarding the use of condensed vs sparse formulations, the different options for QP solvers are discussed underlying the choice of author that consists in using the proximal solvers. Soft constraints are justified and the different formulations are discussed among which the one based on the l_2 penalty is chosen. The chapter ends with a discussion on handling the disturbance via an augmented model and an associated observer.

This chapter is very well written and the condensed style used to underline the essential ideas is really pleasant to read. A significant number of key concepts are conveniently compacted in a short, elegant and clear chapter.

Chapter 3 entitled "*Selected state of the are methods*" firstly describes the Extended Dynamic Mode Decomposition (EDMD) principle with different possible lifting functions for autonomous and controlled dynamics. In the latter case, no control lifting is used. Then the method of eigenfunctions computation is recalled for uncontrolled and controlled (always without input lifting) dynamics.

Here again, the very condensed and still comprehensible presentation of quite technical issues should be highlighted.

Chapter 4 entitled "*Free-variable Koopman predictor (Freeman)*" begins by sketching the main differences in the parameterization of the problem's degrees of freedom justifying the title of the chapter. Then the formulation of the optimization problem leading to the derivation of the linear model in the lifted variables is introduced at a rather high level to provide the *big picture*. The novelty here is to leave every option free and to rely on the optimization to find the solution even if the corresponding problem is non convex. Then, quite technical considerations regarding the respect of state/control **sign-symmetries** that might be present in the original system are discussed inducing a block-diagonal structure for the matrices of the lifted linear system. It is then proved that this does not correspond to a real restriction. On the other hand, a symmetry inclusion condition is imposed on the dataset content as well as the definition of the cost function (by adding appropriate symmetric terms) in order to force the resulting lifting maps to meet the symmetry condition.

Some indications are given regarding the initialization and some tricks about freezing the control input during the first (hundreds of iterations) and recommended descent algorithm using a symbolic computation of the gradient. An unconstrained formulation is used after removing the equality constraints by substitution. After the whole algorithm is summarized, the author underlines some technical and not so obvious issues, namely: The enforcement of the constraints on the inputs rate of change which might be cumbersome given that the control lifting map Ψ is not known a priori, the extension of the definition of the state lifting map and finally the necessary invertibility of Ψ which is needed to return back to the original coordinates for concrete application of the computed control input to the system. For each of these issues, the author enumerates some possible options in an extremely brief manner.

It is not clear at this stage why respecting the symmetry is so important to deserve the place in takes in a so early stage of the method presentation. Indeed, it might be useful to meet the symmetry property but it seems that it comes at a high price in terms of problem-dependent necessary inspection before solving the optimization problem. It might have been useful to delay this presentation as an add-on over a more general and *plug & play* procedure and then to evaluate the differential in terms of computation burden with an without the symmetry exploiting trick. In other words, it is not clear to the reader whether these symmetry-meeting conditions are mandatory for the success of the algorithm or whether they are simply a second order, although very interesting, ingredients that can be added to improve the quality of the derived predictor.

Starting from this symmetry tricks and following the discussion regarding the technical issues and the enumeration of the multiple not so trivial candidate technical solutions that might address these issues, it comes out quite clearly that the problem is much more complicated than one would initially hope. Moreover, it seems quite clear that through all the above mentioned *tricks*, the author's guideline is to keep the **unconstrained character** of the resulting optimization problem which obviously comes with a price in terms of the quality of the resulting solution. The justification of this seemingly in the design of the methodology *hard constraint (namely the one consisting in having ultimately an* **unconstrained** *optimization problem)* is probably dictated by the author's numerical experiments but we, as candide readers, lack discussion and justification and probably facts and figures the author collected during his work.

Chapter 5 shows the application of the methodology sketched in Chapter 4 using some illustrative examples. Three methods are compared, namely the raw EDMD, the optimal eigenfunctions approach and the one proposed by the author denoted as the *Freeman* method. Comparisons in the open-loop and controlled modes show that the proposed approach presents some advantages in terms of allowing discontinuous lifting maps and enabling nonlinear-in-control model to be addressed. Moreover, it shows more robust to changes in the MPC parameters. A comparison is also proposed in terms of computation time with the Casadi-IPOPT multiple shooting implementation showing two orders of magnitude acceleration in favor of the KMPC.

Again, the chapter is well written and the discussion and comparisons are generally interesting and informative. Nevertheless, many aspects should be underlined, namely:

1. The examples contained in this chapter do not really assess the degree of scalability of the approach. Indeed, the first example is scalar, the following two are 2-dimensional oscillators and the third is three dimensional and all of them are open loop stable. Although this scalability problem is not specific to the proposed approach, it remains true that the non convex nature of the proposed solution emphasizes this issue and induces additional reasons to get trapped in a non scalability issue.
2. This chapter lacks some details that might be of interest to the readers and the future researchers on the topics. In particular, it is difficult to know what are the choices operated by the author regarding the many possibilities sketched at the end of chapter 4 to address the problems of rate constraints on the input, the invertibility of Ψ, etc. The readers would have been also interested in the optimization process progress and computation time as well as the sensitivity to the initial guess. Was the process of freezing the control applied in the examples? what is the sensitivity of the success to this trick? etc.
3. The comparison to the nonlinear MPC is rather unfair to me. Indeed, comparing the computation times cannot be decoupled from the quality of the closed-loop cost. This comparison should be done for the same achieved cost, otherwise, one should report both the computation times and the different corresponding cost in pourcentage. A possible comparison would have been to limit the IPOPT maximum number of iterations so that both costs become equal and only then compare the computation times. It is well known that one can limit the maximum number of iterations to 5 or 10 rather than the by-default option and still get very honorable results (that do not rigorously meet the KKT condition to the default precision) that would have been quite comparable in terms of performance to the KMPC while being much faster to compute that reported.
4. Since the author uses the symmetry in the equations in order to derive the specific sparse structures of the linear model's matrices, it seems to be inaccurate to claim that the method is *purely data-based.*

To summarize, I found the author's conclusion: "*KMPC is 100 times faster than NMPC while it is purely data-driven*" rather inaccurate under the light of the previous comments not withstanding the possible domains of application that can be claimed by both methods. Such a conclusion might be, and certainly is, misleading to our community which loves such definitive statements that might (and certainly will) induce dogmatic and erroneous positions.

Chapter 6 concludes part I relative to the Koopman operator.

Chapter 7 entitled "*Preliminaries*" introduces the problem of using the SoS approach to address various control and dynamic systems related problems among which the computation of regions of attraction is addressed in later chapters. The introduction underlines the current issue of computation time and gives the main track that is followed in the next chapters, namely by artificially introduce structure and by splitting the problem into multiple smaller problems.

This chapter is very clear and the use of simple examples to progressively illustrating the transitions between the original problem and its SDP form is a new illustration of the author's remarkable pedagogical skills.

Chapter 8 entitled "*Splitting of the ROA problem*" addresses the SOS formulation of the Region of Attraction Problem. The latter is first formulated in its original form. Then the method of splitting proposed by the author is introduced consisting in piece-wise definition in the time/state space with appropriate boundary conditions to enforce the properties of the original formulation despite of the splitting process. The transformed problem is then written in the SOS form following the rules sketched in Chapter 7. The chapter ends by three examples showing the benefit of the splitting process.

The conciseness that characterized the style of writing of the preceding chapters **reaches here its limit**. Indeed, the author refers to his own published work for further explanations breaking the self-contained character of a PhD manuscript. In particular, the very definition of the optimization problem is not sufficiently linked to the ROA problem. More precisely, the fact that the ROA which is linked to the **existence** of control profiles can be found by a formulation that involves **all possible control trajectories** is not intuitive and would have deserved a longer and patient explanation. Intuitively, this apparent discrepancy in the role of the control should induce the fact that the set founded through this approximation would be much larger than the true one, and this, regardless of the degree of the polynomial being used. But this is probably inaccurate or even wrong! It is unfortunate that the reader is left with such an important question unresolved because of the extreme degree of conciseness adopted by the author in this chapter.

Chapter 9 entitled "*Optimization of the split ROA problem*" pushes the advantage of the splitting method proposed in Chapter 8 by optimization the splitting parameters via first order optimization descent algorithm. As one of the conditions for the existence of the gradient involved in the latter is strong duality, sufficient conditions are proposed to establish strong duality followed by some methods for the estimation/computation of the so provably existing gradient. The different resulting approaches are compared in terms of computation times before the whole process is applied to two examples: the double integrator and the Brockett integrator already seen in the previous chapters.

This chapter is rather technically involved and I found its presentation far less smooth and understandable (when compared to the previous chapters) by readers that are not **fully** specialized in the subject. Regarding the differentiability issue, one might wonder if a derivative-free optimizer could have been used since at the end of the day and as far as the used examples are concerned (the possibility to tackle larger examples is still to be proved anyway), we are talking about few decision variables (of dimension 4, 6 or 8).

Chapters 10 and 11 conclude respectively chapter 9 and the whole manuscript.

Overall assessment and recommendations

The manuscript of Vit Cibulka provides interesting ideas impacting a hot and active topic in the nonlinear control community. His contributions are based on solid and mathematically rigorous foundations and the quality of his hierarchically thoughtful presentation of the ideas is remarkable although pushed too much, in my opinion, in chapters 8 and 9.

Although the scalability and the effectiveness of the proposed approaches on real-life systems and problems remain questionable to me as suggested by the difficulties mentioned by the author himself on rather overly simple examples, the work by Vit Cibulka is, without the slightest doubt, a high quality part of a legitimate and active line of research in our community and the remarkably rigorous efforts and results proposed by the author are necessary to push forward the investigation towards a future commonly shared conclusion.

Based on that, I strongly recommend the PhD defense without the slightest hesitation!

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