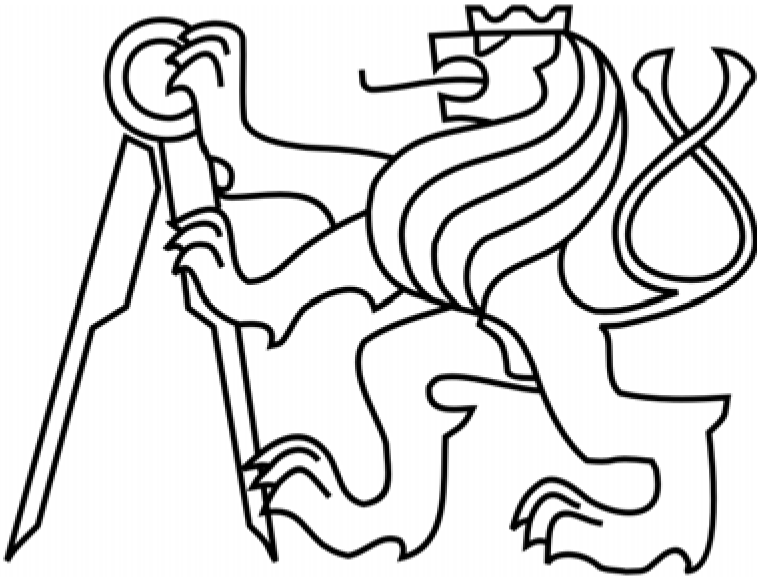
ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

FAKULTA ELEKTROTECHNICKÁ



BAKALÁŘSKÁ PRÁCE

V PRAZE 2008

PAVEL ŠVARC

ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

FAKULTA ELEKTROTECHNICKÁ

KATEDRA ŘÍDICÍ TECHNIKY

Studijní program: Elektrotechnika a informatika
Studijní obor: Kybernetika a měření (bakalářský)

Modelování provozu větrné elektrárny

Wind turbine modeling

Bakalářská práce

Autor:	Pavel Švarc
Vedoucí BP:	doc. Ing. Petr Horáček, CSc.
Oponent:	Ing. Jaroslav Kříž

V Praze dne 25.2.2008

České vysoké učení technické v Praze
Fakulta elektrotechnická

Katedra řídicí techniky

ZADÁNÍ BAKALÁŘSKÉ PRÁCE

Student: **Pavel Švarc**

Studijní program: Elektrotechnika a informatika (bakalářský), strukturovaný
Obor: Kybernetika a měření

Název tématu: **Modelování provozu větrné elektrárny**

Pokyny pro vypracování:

1. Seznamte se s technickými parametry větrné turbíny a generátoru instalovaných na české polární stanici v Antarktidě.
2. Seznamte se s modely pro predikci větrných podmínek vyvinutých pro podmínky ČR v Ústavu fyziky atmosféry AVČR.
3. Navrhněte simulační model elektromechanické části větrné turbíny s generátorem.
4. Navrhněte generátor větru a ověřte funkci modelu větrného turbogenerátoru v takto uměle vytvořených podmínkách .

Seznam odborné literatury:

Dodá vedoucí práce

Vedoucí: doc. Ing. Petr Horáček, CSc.

Platnost zadání: do konce zimního semestru 2008/2009

L.S.

prof. Ing. Michael Šebek, DrSc.
vedoucí katedry

v.z. M. Demlová
doc. Ing. Boris Šimák, CSc.
děkan

V Praze dne 25. 2. 2008

Prohlašuji, že jsem svou bakalářskou práci vypracoval samostatně a použil jsem pouze podklady (literaturu, projekty, atd.) uvedené v přiloženém seznamu.

V Praze, dne ………………………. ……………………………………….

Děkuji doc. Ing. Petru Horáčkovi, CSc. především za důvěru, se kterou mi tento úkol svěřil a za možnost podílet se na zajímavém projektu.

Název práce

Modelování provozu větrné turbíny

Abstrakt

Tento dokument mapuje obecné vlastnosti a chování větru, které jsou následně využity k jeho modelování různými metodami; jmenovitě statistickou metodou zmíněnou v [1] a metodou C. Nichity publikovanou IEEE. Chování větru je modelováno jak krátkodobě, ve smyslu vteřin a minut, tak dlouhodobě jako roční průběhy. Důkladně je analyzována problematika větrných turbín jejíž výsledky jsou použity k vytvoření matematického modelu konkrétní větrné turbíny umístěné na Antarktidě. Výsledky této práce budou využity jako součást řídícího systému produkce elektrické energie v místě umístění turbín.

Klíčová slova: energie, model, vítr, větrná turbína.

Title

Wind turbine modeling

Abstract

This paper describes behavior and general properties of wind. Given background is then used for wind speed modeling by different methods; namely statistical method mentioned in [1] by Manwell and C. Nichita's method published by IEEE. Wind behavior is being modeled in short time scale, in terms of seconds and minutes as well as in long time scale as year patterns. Problematic of wind turbines is being analyzed in depth which is followed by modeling of particular wind turbine located in Antarctica. Outcome of this paper will be utilized as a component for a control system of energy production at the site of the modeled wind turbine.

Keywords: energy, model, wind, wind turbine.

List of abbreviations:

HAWT	Horizontal-Axis Wind Turbine
VAWT	Vertical-Axis Wind Turbine
pdf	Probability density function

List of symbols:

U	wind speed	[$m \cdot s^{-1}$]
μ, $\bar{U}$	mean wind speed	[$m \cdot s^{-1}$]
β_w	Weibull shape parameter	[-]
η_w	Weibull scale parameter	[-]
γ_w	Weibull location parameter	[-]
m	Mass of air	[kg]
P_w	power stored in the wind	[W]
P_T	power produced by the wind turbine	[W]
ω	angular velocity	[$rad \cdot s^{-1}$]
C_p	power coefficient of the wind turbine	[-]
C_{pmax}	maximal power coefficient	[-]
TSR	tip speed ratio	[-]
β	pitch angle	[°]
A	area swept by blades of the wind turbine	[m^2]
R	radius of the wind turbine	[m]
J	moment of inertia of the wind turbine	[$kg \cdot m^2$]
T_T	torque of the wind turbine	[$N \cdot m$]
T_G	load Torque of the generator	[$N \cdot m$]
η_{Gear}	efficiency of the gearbox	[-]
η_{Gen}	efficiency of the generator	[-]
p	pressure of air	[Pa]
t	time	[s]
ρ	density of air	[$kg \cdot m^{-3}$]
Ke	energy pattern factor	[-]
$\bar{U}_m$	average wind speed in month m	[$m \cdot s^{-1}$]
σ_m	standard deviation of wind speed in month m	[-]
$\bar{U}_{m,h}$	average wind speed in month m and hour h	[$m \cdot s^{-1}$]
$\sigma_{m,h}$	standard deviation of wind speed in month m and hour h	[-]

Table of Content

1. Introduction

1.1 Motivation and background: Antarctica project

Antarctica belongs to a very tiny group of sites which are almost not influenced by humans and their activities. That is the reason why Antarctica becomes more and more popular among scientists of any kind – it is an example of pure wildlife. Several countries have their scientific stations located at the most southern continent of the world with their teams observing the treasures of nature which have not been seen before. The Czech Republic after years of preparations built its own scientific station in the northern part of James Ross Island in 2006 with multiple targets and plans summarized in [14]. Every station built in Antarctica has to minimize its waste production and fossil fuels consumption to maintain this region undefiled. Therefore the primary sources of energy are renewable energy sources – wind and solar radiation. It has to be noted that these energy sources are not always available – their power output is not guaranteed (e.g. wind does not flow with the same velocity or clouds cover the sun). To achieve the best results of energy production a sophisticated control has to be applied to combine, in the best possible manner, sources of energy in relation to their actual possible output. This document is focused on two important components for developing such control system: model of the wind and model of the windmill.

1.2 Goals formulation: Wind and windmill modeling

The main goal of this paper is to create model of the wind and model of the windmill. For derivation of these models physical background will be given in Part I where wind will be described as a source of energy and windmill and its components will be presented.

Different approaches and methods of the wind modeling will be introduced and applied on the data of wind speeds in the Czech Republic in Part II. There will be also developed mathematical model of the turbine utilized in Antarctica using data from the manufacturer.

Model of the wind from Part II will be used in Part III as an input for the model of the windmill. Results will be then discussed in the very same chapter.

The results of the work described above will be also presented in the interactive demo where the simulation, both the model of the wind and the windmill, will take place. Appendix D – DEMO will be devoted to the functions and control of the demo.

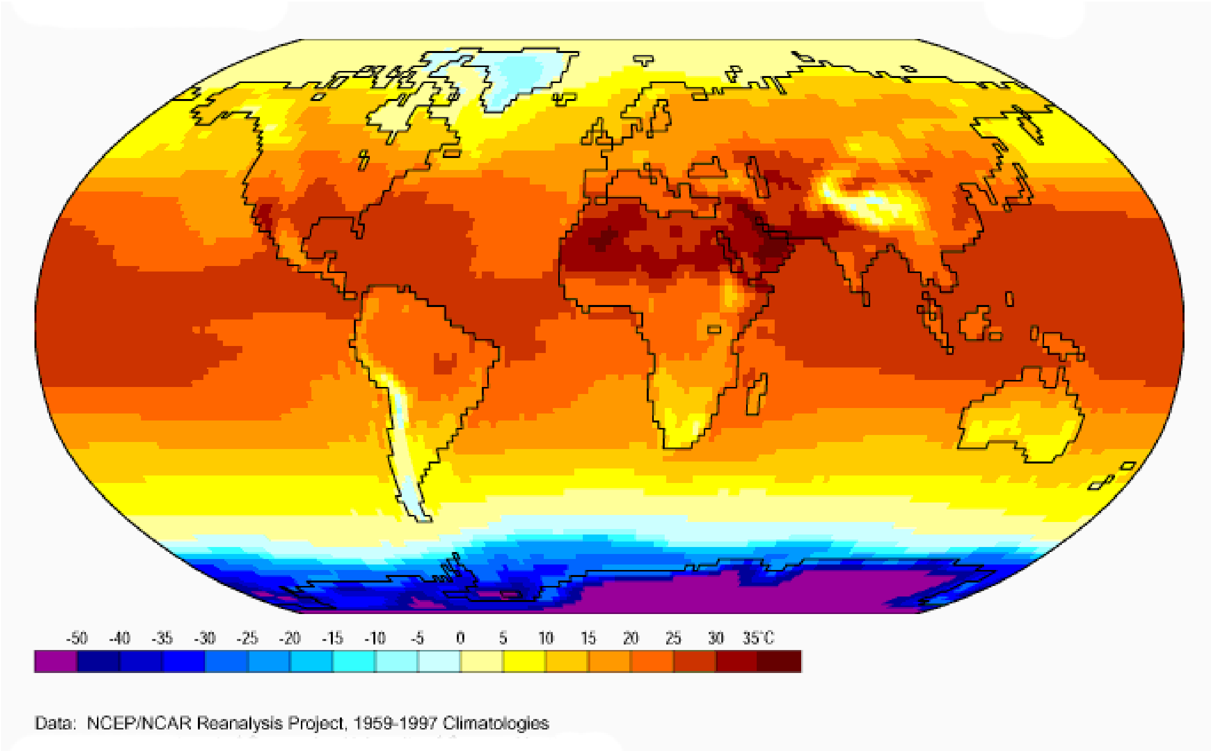
2. Part I: Wind & Windmills

2.1 Wind and its properties

In this chapter principles of wind behavior over large and small scale (in terms of time and space) will be summarized. This information will be later in this paper used for wind modeling.

Global winds: Power of the sun

"All renewable energy (except tidal and geothermal power), and even the energy in fossil fuels, ultimately comes from the sun. The sun radiates 174,423,000,000,000 kilowatt hours of energy to the earth per hour. In other words, the earth receives 1.74 x 10^{17} Watts of power. About 1 to 2 per cent of the energy coming from the sun is converted into wind energy. That is about 50 to 100 times more than the energy converted into biomass by all plants on earth."[1]



Data: NCEP/NCAR Reanalysis Project, 1959-1997 Climatologies

Figure 2-1: World map of air temperatures[2]

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[1] [13], Guided tour>Wind>Whence a wind?

[2] [18]

The figure above shows that heating the air by sun radiation is the highest at latitude 0° and gradually decreases while approaching the poles. This phenomenon has the effect of air circulation – hot air raises up and moves towards the poles. Since Earth rotates so called Coriolis force occurs. This force is in detail explained in [4] and has the effect illustrated at the figure below. Wind due to high and low pressure areas ascends and descends. Total effect in global wind directions is summarized in the table below.

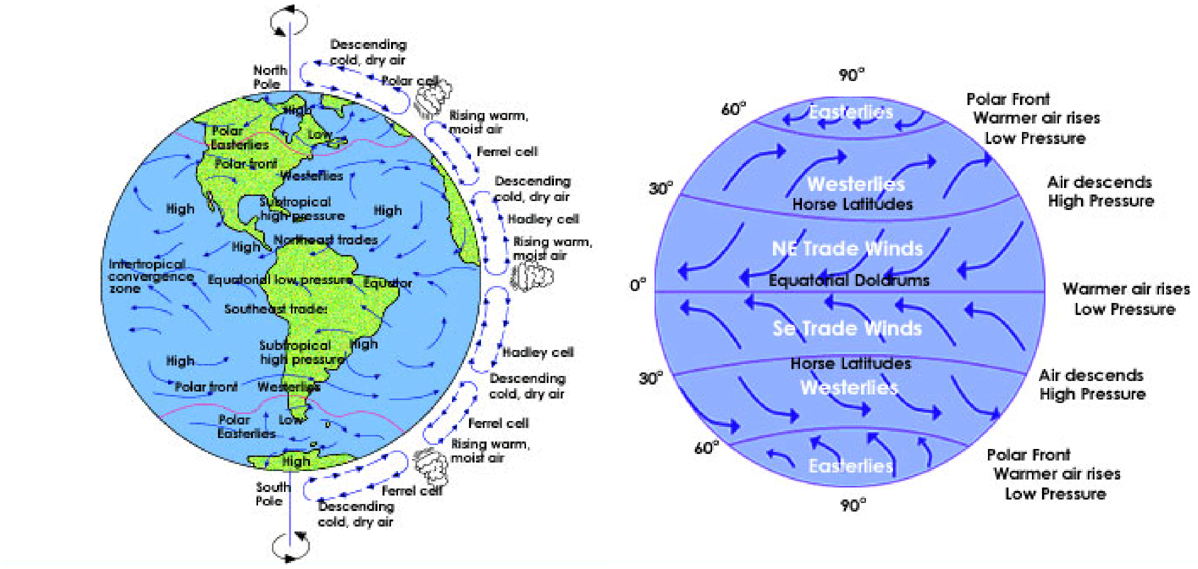

Figure 2-2: Directions of wind and high and low pressure areas[3].

Latitude	90-60° N	60-30° N	30-0° N	0-30° E	30-60° E	60-90° E
Direction	NE	SW	NE	SE	NW	SE

Table 2-1: Summary of wind directions at different latitudes.[4]

Wind does not have to strictly follow directions in Table 2-1, locally it is influenced by the roughness of ground's surface and its shape. These local winds are used for generating energy by windmills and will be further discussed in the next chapter.

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[3] [19]

[4] [13], Guided tour>Wind>Global Winds

Local winds: Effect of terrain

Shape of terrain over which wind flows has important impact on the wind characteristics as it was mentioned above. "The most basic classification of terrain divides it into flat and non-flat terrain."[5] Non-flat terrain is divided into two subcategories: isolated elevation and mountainous terrain. In classification of terrain has to be taken in account whether the obstacles have direct impact on the proposed site of the windmill (e.g. high mountain could be 100 m from the proposed site but wind from this direction blows extremely rarely therefore this terrain should be classified as flat in spite of its shape). Considering that wind with the lowest possible turbulence component (for description of wind components see 2.1: Wind statistics and patterns) is desired deeper description of wind behavior over non-flat terrain which causes turbulences will be left out.

Flat terrain along [1] is depicted by the following:

- There is not elevation difference greater than 60 m anywhere in the circle with diameter of 11.5 km around proposed windmill site.
- No hill has an aspect ratio (height/with) greater than 1:50 within 4 km upstream and downstream of the site.
- The elevation difference between the lower end of the rotor disk and the lowest elevation of the terrain is greater than three times the maximum elevation difference (h) within 4 km upstream. (See figure below)

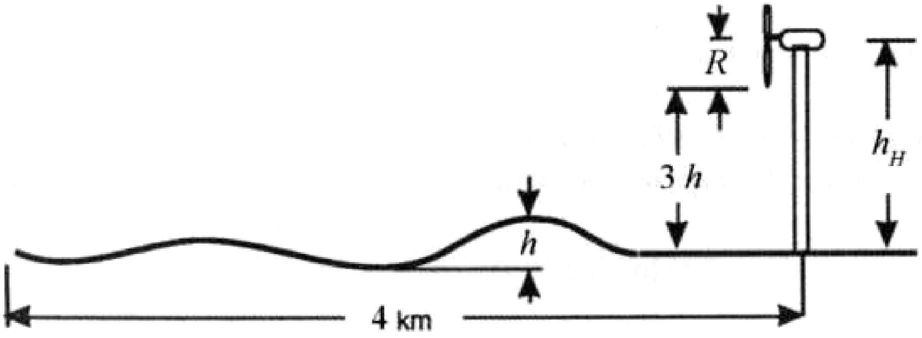

Figure 2-3: Flat terrain characteristics along [1].

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[5] [1], p.46

Obstacles in flat terrain can be human made (buildings, silos etc.) or natural (lines of smaller trees etc.). Influence of these obstacles on the wind's behavior while flowing over them is important phenomenon for determining site's suitability for a wind turbine. 2D illustration of wind characteristics flowing over an obstacle with blockish shape (e.g. building) is shown on the figures below.

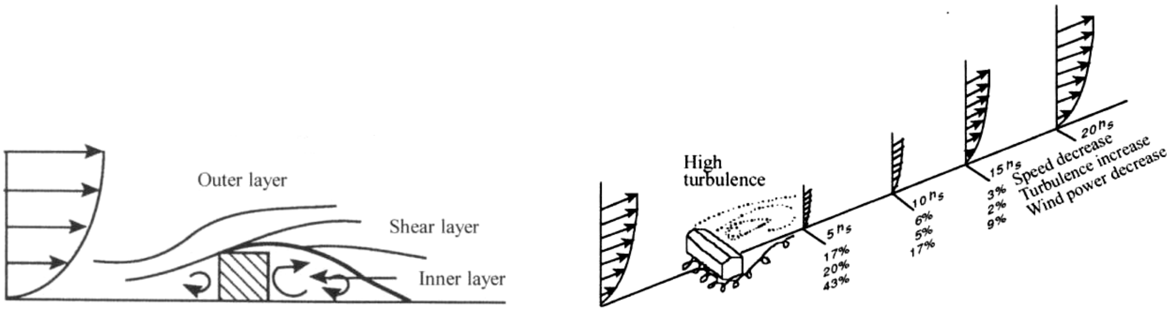

Figure 2-4: Wind flowing over an blockish obstacle along [1] p.47.

From the Figure 2-4 it is clear that the influence of the obstacle of height *h* on the wind behavior (wind speed and direction) is minimized in the distance of 15 *h*.

The most desirable site for windmill as it was shown is as flat terrain as possible. There are smallest power losses (wind is less turbulent and has higher speed). Non-flat terrain can also have very good potential for windmills, for example in canyons or narrows where wind is naturally forced into smaller volume and therefore increases its speed (tunnel effect). This type of behavior is much more complex and was left out from this study. There is also an influence of the "roughness" of the terrain which is called shear. There is a measure called roughness class[6] which determines how much is wind being slowed down by the terrain (for example region with many trees and buildings has roughness class 3; airport runway 0.5). The slowdown of the wind due to the terrain decreases with height above the ground. For detailed study of shear see [13] or [1], p.47.

Not only wind speed is essential for deciding whether proposed site is suitable for a wind turbine or not, wind direction has to be also studied. Wind direction is often illustrated in special type of graph shown below called wind rose which graphically express from what direction flows wind and how much this direction changes through the year.

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[6] [13], Guided tour>Turbine siting>Roughness & shear

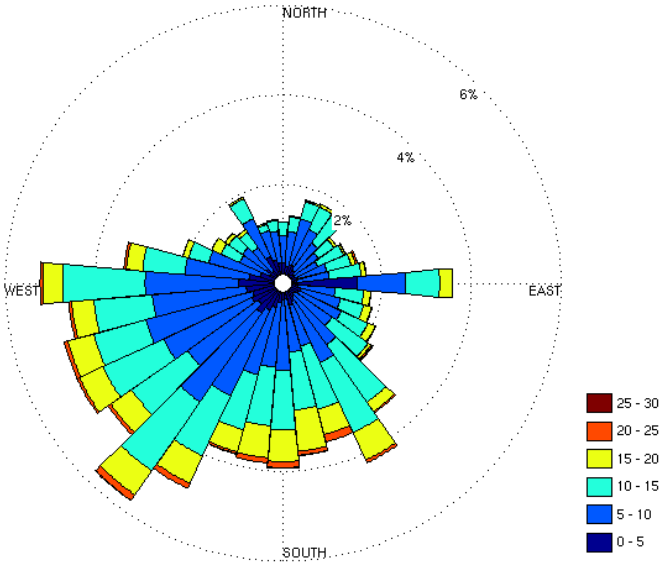

Figure 2-5: An example of wind rose.[7]

For optimal power production it is necessary to ensure that the wind flow is perpendicular to the rotor. This is achieved by yawing the turbine on the run (while the rotor is rotating). Short-term changes in the wind direction (turbulences) are not desirable because of the speed limit of yawing. Yawing on the run especially for bigger windmills is quite challenging task because of enormous gyroscopic loads (for deeper analysis of gyroscopic load see [1], p.148). Therefore the short-term wind direction is desired to be as stable as possible. Desired is also long-term wind direction stability because yawing the turbine causes fatigue of the material since there are huge loads as it was mentioned before.

Wind statistics and patterns.

Wind and its behavior in time is being described by many various methods. This chapter focuses on the wind description by finding patterns in wind behavior by statistical analysis. In this text background needed for wind modeling by different methods will be given and selected methods will be described in Part II.

For start patterns in the wind behavior can be divided into several categories along their variation in time[8]:

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[7] [20], Specialized Plot and Graph Types

[8] [1], p.26

- **Inter-annual:** These variations occurs over a time period greater than one year and have significant impact on the overall power production of a wind turbine. Meteorologists generally conclude that it takes about 30 years of data to find out inter-annual variations.
- **Annual:** Considerable patterns in month mean wind speeds occur for almost all sites. For example in the Czech Republic in winter months mean wind speed is much higher than in the summer.
- **Diurnal:** On daily time scale there is also a pattern in wind speeds which is caused by sun radiation – heating and cooling of the ground. In seasons when night and day temperature differences are not very significant diurnal pattern can be hardly seen, but for example in summer when day temperatures strikes high values and night temperatures falls rapidly diurnal variation is strong.
- **Short-term (gusts and turbulences):** As short-term variations are meant changes in wind speed over a period less than 10 minutes. These phenomena are called turbulences and gusts. Turbulence has stochastic behavior which is based on the mean wind speed (the higher mean wind speed is the more turbulent wind is). "Gust is discrete event within a turbulent field"[9]. Its rise time, amplitude, gust variation and lapse time can be observed.

Particular wind speeds are distributed over a mean wind speed with given probability density function (pdf). In this text two different distributions will be presented and described: Rayleigh distribution and Weibull distribution.

Rayleigh probability distribution is the simplest type of distribution function used for phenomena such as wind speed variation. To calculate probability *p(U)* of particular value *U* only mean value *μ* is required. For derivation of the following equation see Appendix B – Statistics.

$$p(U) = \frac{\pi}{2}\left(\frac{U}{\mu^2}\right)e^{\left[\frac{-\pi}{4}\left(\frac{U}{\mu}\right)^2\right]} \qquad (2\text{-}1)$$

Probability density functions for several different mean values are displayed on the figure below.

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[9] [1], p.29

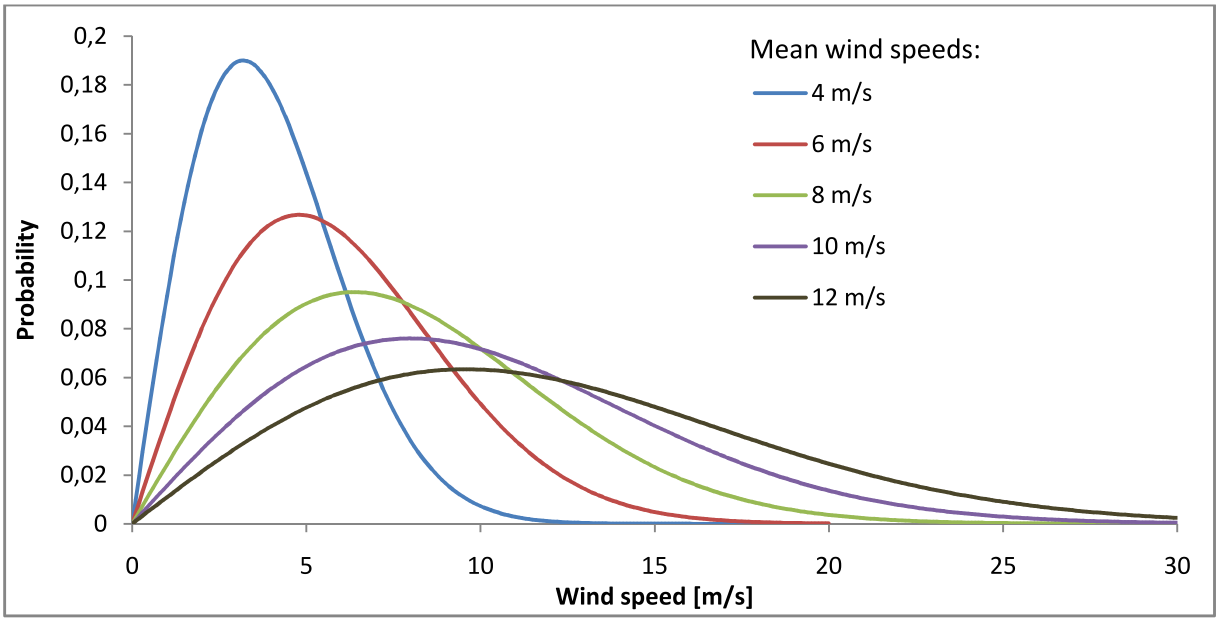

Figure 2-6: Rayleigh distribution for several mean wind speeds.

Weibull distribution is more complex than Rayleigh distribution, it requires from one to three parameters for determining its shape[10] and its pdf is given by:

$$p(U) = \frac{\beta_w}{\eta_w}\left(\frac{U-\gamma_w}{\eta_w}\right)^{\beta_w-1} e^{-\left(\frac{U-\gamma_w}{\eta_w}\right)^{\beta_w}} \qquad (2\text{-}2)$$

Where β_w is shape parameter, η_w is scale parameter, γ_w is location parameter.

Using Weibull distribution almost any shape of probability density function can be obtained. Parameters of Weibull distribution are similar for countries in the same geographical region. Figure below shows several shapes of Weibull pdf with different parameters and can be obtained from measured data. Formulas for calculation of these parameters with other complex information about Weibull distribution are located at [12].

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[10] [12], The Weibull Distribution

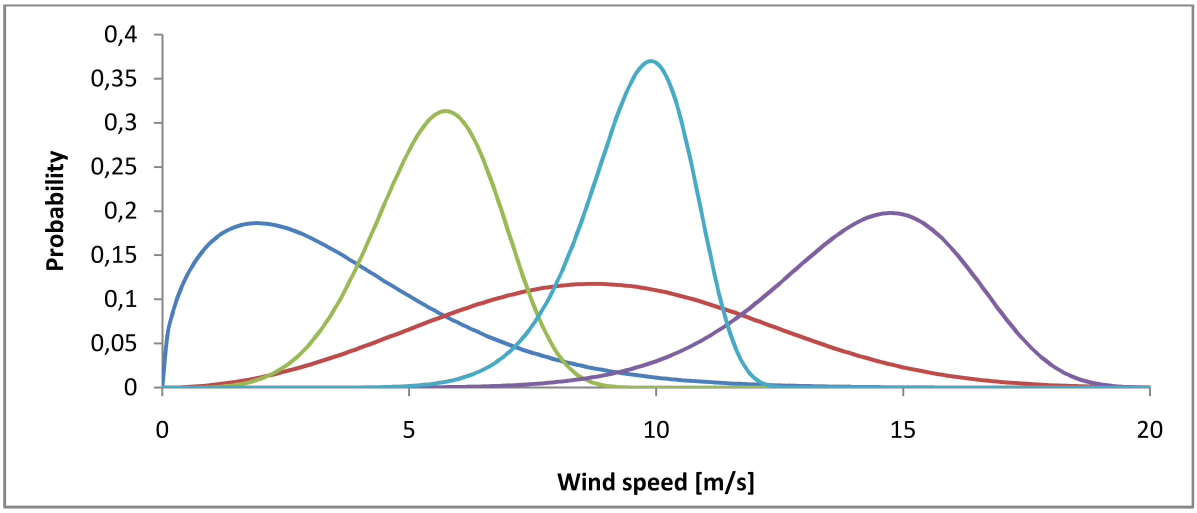

Figure 2-7: Weibull distributions with different parameters.

Particular wind speeds are not only distributed along a specific function, they are also time-correlated. By time-correlation is meant a percentage of how much can following wind speed in chosen time scale differ from the value previous. For example wind speed in time *t* is 10 m·s^{-1} and in time *t+1* values of wind speed can vary from 7 to 13 m·s^{-1}, no more or less. Wind simply cannot change its speed infinitely fast; there is a limit of its rate of change. This limit varies with the mean wind speed along which it is distributed – the faster wind blows the faster its speed can change. Method of finding this dependence by autocorrelation is described in [1], p.40.

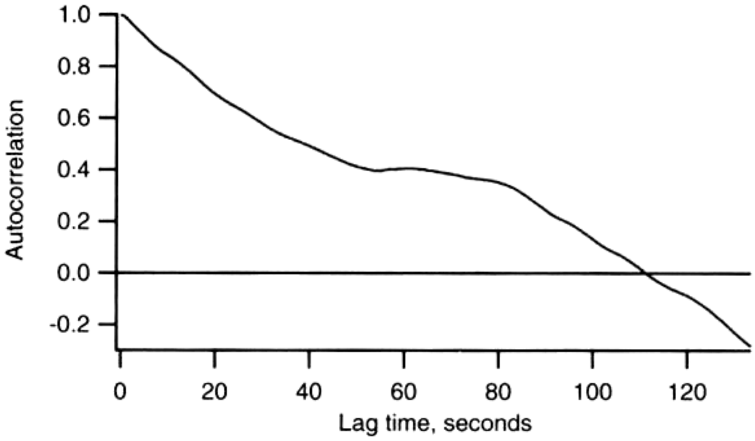

Figure 2-8: Autocorrelation of wind speeds. The bigger time lag between measurements is the less are wind speeds correlated.

This chapter described the nature of the wind and its properties, which will be utilized for wind modeling in Part III.

Wind behavior: Passing through the turbine (HAWT[11])

Wind turbine composed of specific number of blades (from 2 to 6 or rarely more) acts as an obstacle to the wind changing its properties. For start few reasonable assumptions have to be made. Wind flowing towards the turbine is perpendicular to blades and is uniform (has equal speed). The very first idea is that the wind flows through the turbine like in a tube as it is presented in the figure below. There are many arguments that deny this concept.

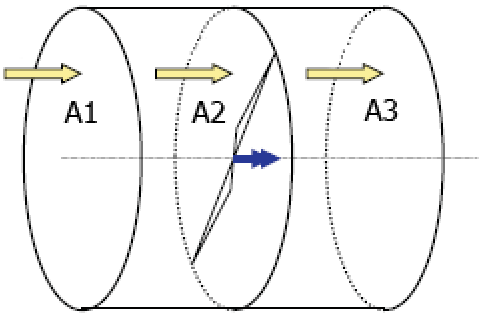

Figure 2-9: Tube concept of wind flowing through the turbine.[12]

Wind approaching the turbine slows down and tries to avoid it similarly as the obstacles in the previous chapter. Therefore pressure increases until the turbine is reached and after passing it pressure falls down significantly. Turbine extracts energy from the passing wind and rotates. This energy loss of the wind has the effect on the wind speed which decreases. Since the same amount of air enters the turbine and the very same amount leaves the turbine each time period and the speed of entering wind is higher than the speed of the wind leaving the turbine, air would have to cumulate behind the turbine (let say the turbine is 100% effective – all energy from wind is taken – wind would have to stop and cumulate after passing the turbine which is not possible). Air does not cumulate but spreads to area larger than the diameter of the turbine, the cross sectional area of the wind "tube" increases. Wind behavior which was just described is illustrated by the figures below.

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[11] Horizontal-Axis Wind Turbine. Types of turbines will be presented later in chapter 2.2

[12] [5], p.51

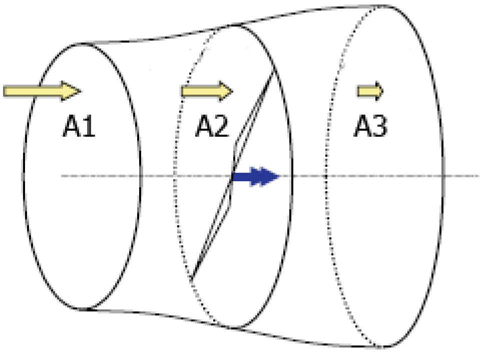

Figure 2-10: Increase in volume of air from *A1* to *A3* after passing through the turbine (arrows represent value and direction of wind)[13].

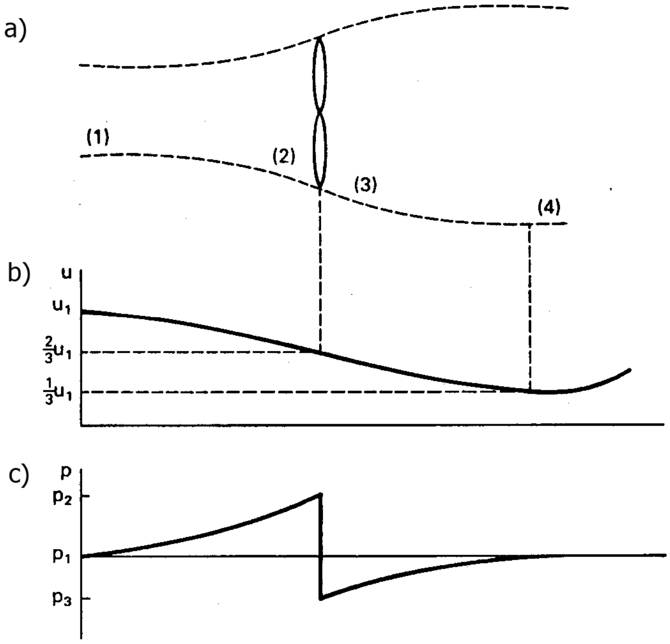

Figure 2-11: a) Change in the diameter of a wind „tube". b) Change in the wind speed. c) Increase in air pressure *p1* to *p2* while approaching the turbine and sudden drop to *p3* after reaching the turbine[14].

The phenomenon described above would prompt that the measuring the wind speed before entering and after leaving the turbine would give us pretty accurate result of extracted energy from the wind, but there are another influence of the wind turbine on the wind's behavior. For example rotation of blades gives part of theirs torque to the wind which creates a wake behind the turbine. Wake is documented on the figure below.

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[13] [5], p.51

[14] [7], p.3

Figure 2-12: An experiment which demonstrates the rotational behavior of the wind passing through the turbine (wake) and also the change of the volume of air leaving the turbine (smoke spreads away from the turbine).[15]

This chapter summarized behavior of the wind in the direct contact with the turbine. Explanation and figures in this text will be used in the next chapter for calculation of energy trapped in the wind and its possible extraction. Wind behavior while it is approaching and passing through the turbine is more complex than is was presented in the text above. There are several other complicated processes which were omitted from this paper.

Energy in the wind

The aim of this chapter is to find how much energy is stored in the moving air and how much of this energy can be taken by the windmill.

Let's consider a circular cross sectional area *A* having diameter equal to a diameter of turbine's rotor. In time *dt* a mass of air *dm* flows through *A*. This can be written in the following formula.

$$\frac{dm}{dt} = \rho A U \qquad (2\text{-}3)$$

Where ρ [kg·m^{-2}] is air density (1.225 kg/m^2 at sea-level, 15°C), U [m·s^{-1}] is the air velocity.

Power of the flowing mass (or kinetic energy per unit time) through area *A* is given by:

---

[15] [4], p. 711

$$P_w = \frac{1}{2}\frac{dm}{dt}U^2 = \frac{1}{2}\rho A U^3 \tag{2-4}$$

Noteworthy is the cubic dependence of the power output on the wind speed; its importance will be pointed out later in the text.

When data including hour average wind speeds of a certain site are available, useful information can be calculated saying if this site is suitable for windmill or not along [1], p.32. First of all power per unit area has to be stated:

$$\frac{P_w}{A} = \frac{1}{2}\rho U^3 \tag{2-5}$$

Then the mean power production per unit area based upon hourly means is given by:

$$\frac{\overline{P_w}}{A} = \frac{1}{2}\rho \bar{U}^3 K_e \tag{2-6}$$

Where $\bar{U}$ stands for annual mean wind speed and K_e is so called energy pattern factor which is given by:

$$K_e = \frac{1}{N\bar{U}^3}\sum_i^N U_i^3 \tag{2-7}$$

Where N is number of hours in a year (8760) and U_i is measured hour mean wind speed.

There qualitative criteria of sites which are proposed for windmills along [1]:

- $\bar{P}/A$ < 100 W/m2 - poor
- $\bar{P}/A$ ≈ 400 W/m2 - good
- $\bar{P}/A$ > 700 W/m2 - great

Conclusion

Chapter 2.1 gave us sufficient background of wind characteristics and its behavior. Using this information it is possible to workout methods of wind modeling and prediction which will be shown in Part III.

2.2 Windmills and their components

Introduction

Chapter 2.2 focuses on the windmills problematic. Components of the windmills will be presented and their functions and behavior described. The main goal of the following text is to get sufficient physical background to be able to create a model of given windmill in Part II.

Overview

Windmill is a machine which takes mechanical energy (kinetic energy) from the wind and converts it into the electrical energy. There are two general types of windmills HAWT (Horizontal-Axis Wind Turbine) and VAWT (Vertical-Axis Wind Turbine). Windmills can be also divided into several categories along their position or power output which can vary from few kilowatts to several megawatts.

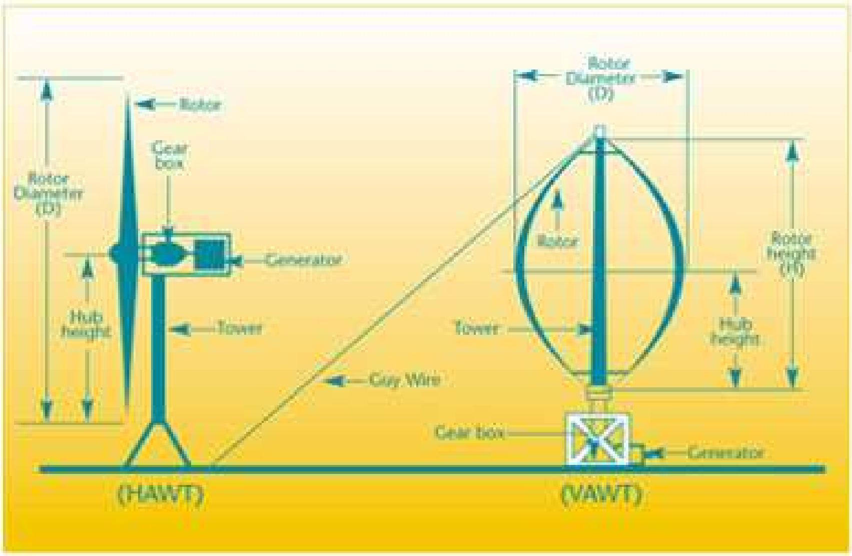

Figure 2-13: HAWT and VAWT with description of their components[16].

This paper is focused only on HAWT because windmills of this type are installed in Antarctica.

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[16] [17], 2008>January>Vertical-Axis Wind Turbines

Windmill is composed of rotor which has two or more blades, gearbox and generator. In case of small turbines there could be a tail for turning (yawing) the whole turbine into the direction of the wind to maximize its power production.

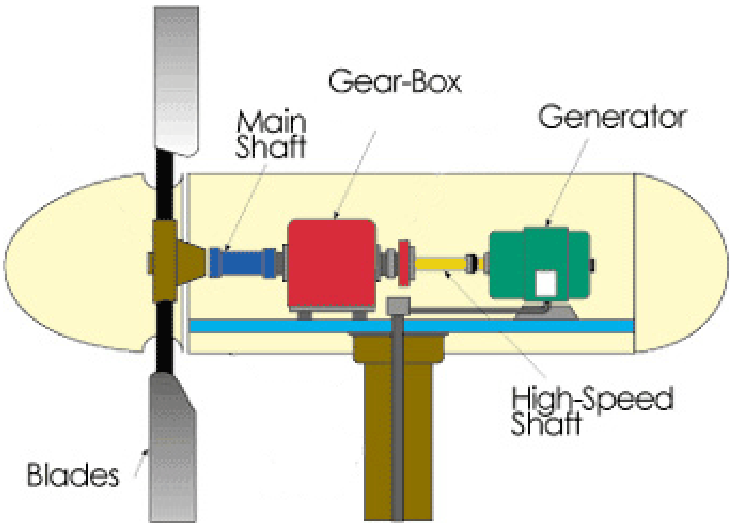

Figure 2-14: Components of standard windmill[17].

Power extraction

The amount of energy in the flowing air was stated in the chapter 2.1. Only part of this energy can be taken and transformed by the wind turbine. This percentage is represented by the coefficient C_p in the following formula derived in the chapter 2.1.

$$P_T = \frac{1}{2}\rho A U^3 C_p(TSR, \beta)\eta \quad (2\text{-}8)$$

Coefficient η stands for efficiency of other windmill components such as gearbox or generator.

There is a certain limit of the windmill effectiveness. Top theoretical value is called Betz's constant and it is equal to $C_{pmax} \approx 0.59$. Derivation of this value is documented in the Appendix A – Physics. Windmill effectiveness is dependent on the tip speed ration (*TSR*) and the pitch angle (in the case when angle of the blades is adjustable). Tip speed ratio is given by:

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[17] [21], Sustainable Energy>Wind

$$TSR = \frac{R\omega}{U} \tag{2-9}$$

R [m] is the rotor radius, ω [rad·s^{-1}] is the angular velocity of the rotor and *U* [m·s^{-1}] is the wind speed. *TSR* is one of the most important parameters of design of a windmill. Usually up to certain point the higher *TSR* is the higher C_p is, *TSR* often varies from 5 to 10[18].

Possible characteristic of C_p as a function of *TSR* and β (or for constant β) is shown on the graphs below.

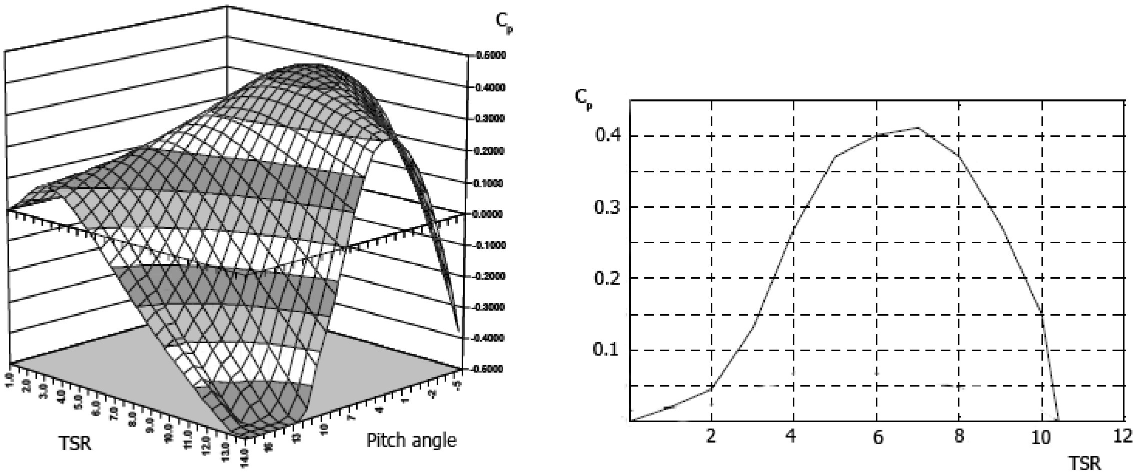

Figure 2-15 a) C_p as a function of Pitch angle and TSR[19]. b) C_p as a function of TSR[20].

Unique characteristics shown above of a specific wind turbine (which are determined mainly by the shape of the blades) are necessary for developing the dynamic model of the turbine as it will be shown in Part III.

Rotor: Dynamics of blades

Rotor "catches" part of the kinetic energy of the flowing wind by slowing it down and transforms it into angular velocity of the rotor shaft. Shape of the rotor, respectively shape of the blades (airfoils) determines effectiveness of the turbine. Turbines are usually three bladed, it is a compromise: two bladed turbines have much lower effectiveness and on the other hand four bladed turbines are too expensive to manufacture. There are plenty different

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[18] [4], p. 12
[19] [8], p.2
[20] [10], Development of a Wind Turbine Simulator for Wind Generator Testing

shapes of blades from various materials having only one purpose: maximize power extraction from the wind.

As it was mentioned in the previous chapter there is a certain limit stating maximum percentage of energy which can be taken from the wind. This value is only theoretical and it can never be reached in practice. Effectiveness of a windmill varies from very low numbers for home made turbines and hardly ever reaches 0.5. It has to be said that C_p varies with the wind speed (it is a function of *TSR* which is a function of wind speed). Graph below on the left shows comparison of energy of the wind, maximum theoretical amount of energy which can be taken (limited by Betz's constant) and energy taken by a standard windmill. Graph on the right shows how C_p varies with the wind speed (characteristic of typical Danish windmill with average efficiency about 20%)[21].

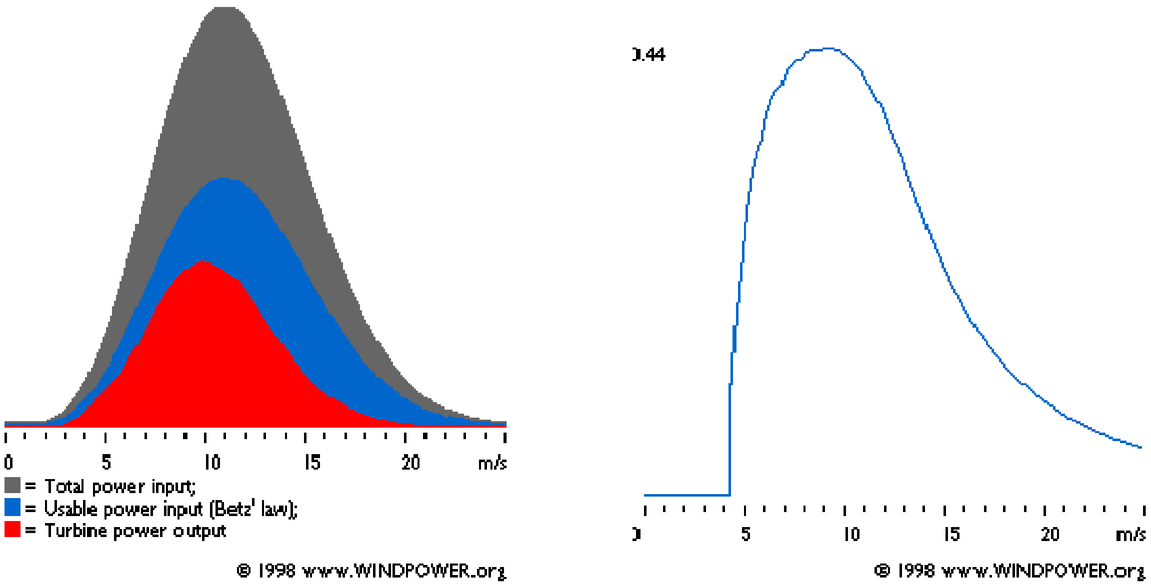

Figure 2-16: a) Power density functions. b) C_p as a function of the wind speed.

Each wind turbine has its cut-in and cut-out wind speed. These speeds determines when turbine starts to produce energy (cut-in speed, varies usually between 3-5 m/s) and when is taken out of order by turning the rotor out from the direction of the wind or simply applying a brake to prevent its damaging (cut-out speed, around 25 m/s). Influence of cut-in and cut-out speeds has the effect of chopping the usable power (blue curve in the Figure 2-13 a) on the left at cut-in speed and on the right at cut-out speed. Value of cut-in speed is very clear

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[21] [13], Guided tour>Energy output>Power density

from the Figure 2-13b, where efficiency is 0 until reaching this speed. Cut-out speed would be illustrated as a sudden drop of C_p to 0.

Since general characteristics and properties of the wind rotor were described the shape of the blades and its influence on the dynamic behavior of the rotor can be briefly presented.

Air flowing over an airfoil produces two forces: lift and drag. Lift force causes rotor to rotate therefore it is desired to be as high as possible; on the other hand drag force is undesirable, because it acts in the direction of the wind - as a load on the wind turbine's tower. Directions of these forces are illustrated below.

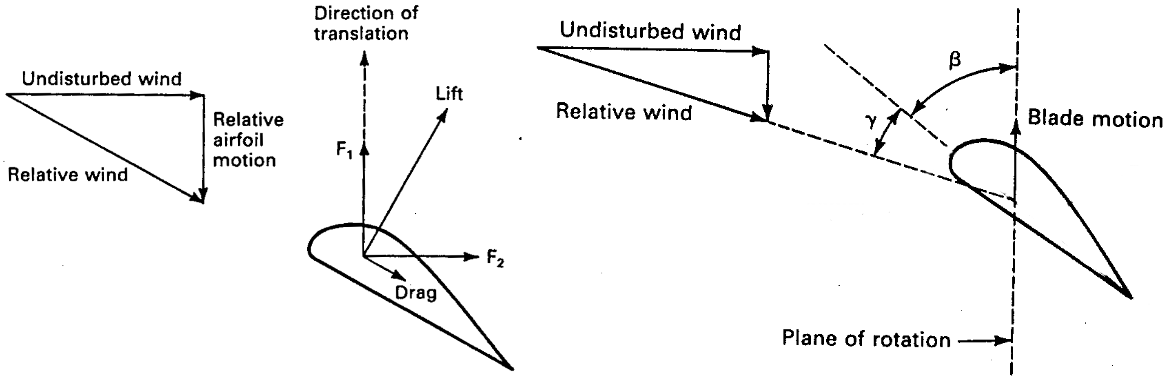

Figure 2-17: a) Lift and drag forces (splitted into components F_1 and F_2 perpendicular and parallel to the direction of rotation). b) Pitch angle (β) and angle of attack (γ)[22].

The reason why lift occurs is change in pressure. Because of the shape of the airfoil wind on the upper side has longer distance to travel than the wind on the lower side, therefore it speeds up. This effect can be described by Bernoulli's law[23]:

$$P + \frac{1}{2}\rho v^2 = constant \qquad (2\text{-}10)$$

The first term P is static pressure and second term is dynamic pressure. Since the wind speed v changes there has to be corresponding change in static pressure. This change in pressure (lower pressure on the upper side of the airfoil) causes rotor to rotate.

There are two important blade parameters: pitch angle and angle of attack (figure 2-17b). Pitch angle is a static angle between the blade and hub determining the orientation of the blade. Angle of attack influences values of lift and drag coefficients – to a certain limit of

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[22] [7], p.5-6

[23] [4], p.14

angle of attack lift force increases (bigger force is applied on the rotor in the direction of rotation) but also drag force increases (acts negatively on the whole turbine). After reaching this angle lift force drops and only drag force acts on the rotor. Angle of attack depends on both speed of the blade and speed of the wind. There is only one ideal angle of attack for desired drag and lift coefficients, therefore to maintain this optimum value along whole blade twist is built in. Possible characteristics of lift and drag forces are presented below.

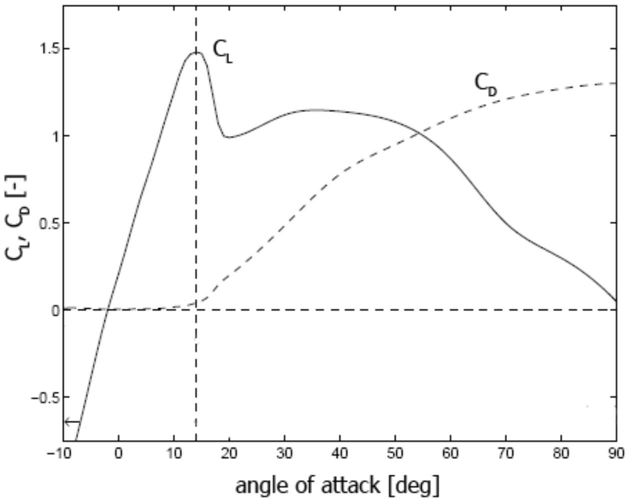

Figure 2-18: Lift (C_L) and drag (C_D) coefficients as a functions of angle of attack.

Lift and drag coefficients along with other blade properties are responsible for the value of the power coefficient C_p. For stating these coefficients special software is usually used where a library with many types of airfoils is included. Coefficients of different airfoils are obtained from wind tunnels where airfoils are being tested.

Gearbox

Usually the generator of a windmill is manufactured for much higher rated rotor angular velocity than the angular velocity of the turbine rotor – turbine rotor would have a tip speed greater than the speed of sound if it were to accommodate a standard generator[24] (this assumption is made for generators of huge wind turbines with power output of hundreds of kW or several MW). Gearbox changes the shaft speed, by the different number of teeth on the gears. The gear connected to a turbine rotor (low-speed shaft) has more teeth than the gear connected to the high-speed shaft connected to the rotor of the generator. The effect is

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[24] [13], Guided tour>Generators>Gearboxes

that the gear with lower number of teeth has increased speed and decreased torque. Total power *P* remains same for ideal gearbox as it is shown in the formula below.

$$Q_1 \Omega_1 = Q_2 \Omega_2 = P \tag{2-11}$$

Where Q_1 stands for torque and Ω_1 is the angular velocity of one shaft and similarly Q_2 and Ω_2 for second shaft. The ratio between the angular velocities of two shafts is inversely proportional to the ratio of the numbers of teeth of gears[25]:

$$\Omega_1 / \Omega_2 = N_2 / N_1 \tag{2-12}$$

Inertias and frictional forces of the shafts and gears are neglected but at greater systems these should be calculated along standard principles of kinetic energy. Dynamics of the gearbox is described in [15], p.21.

It was assumed that the power *P* transmitted by the gearbox remains constant, but there are some losses of energy due to the friction. Efficiency of the gearbox is stated by the percentage η_{Gear} included in coefficient η in equation (2-8). Small windmills with low power output (up to 10 kW) do not have a gearbox and turbine's rotor is directly connected to the generator. Gearbox can be also utilized as a part of control system – changing the gear on the run adjusts the rotor speed of the generator and could help to maximize power output of the windmill.

Generator

Generator converts mechanical power (kinetic energy of rotating shaft) into electrical energy which is then transported into the grid or to batteries. There are several types of generators used in windmills. Two main categories are synchronous and asynchronous (induction) machines. In this paper both synchronous and asynchronous machines will be briefly described, however the main focus will be on synchronous generators since these are used in windmills located in Antarctica.

Nowadays majority of wind turbines all over the world use asynchronous generators. There are many advantages in using them, namely their price and reliability. Asynchronous machines can be connected directly to the grid without any sophisticated and expensive

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[25] [1], p.147

additional devices. Windmills which are using this type of generators are usually not stand-alone since AC current is needed for start-up, therefore grid connection is often used.

Synchronous generators are more expensive than asynchronous machines and also their utilization is more complicated but they are more efficient. Synchronous generators can be further divided into two categories: with permanent magnets or with a DC link. Large turbines usually have a DC generator connected to the shaft to generate DC current to fed synchronous generator of the turbine. Generators with permanent magnets does not need any external source of energy, therefore they are often stand-alone. For connecting synchronous generator to the grid there has to be a control of angular velocity of the rotor since the frequency is directly dependent on it. There are several methods how to make the frequency constant. Usually a combination of mechanical (change in pitch angle) and electrical (change in feeding DC voltage) methods are used.

Efficiency of a generator is stated by the percentage η_{Gen} included in coefficient η in equation (2-8). Further information about synchronous and asynchronous machines is in [1], p.197 or in [22]. In the next chapter possibilities of storage or transport of generated power will be discussed.

Connection to grid or to battery

There are several methods how can be the energy from the windmill transported to the consumer. In this chapter these methods will be described and their efficiency discussed.

Generator which is directly connected to the grid has to produce AC at constant frequency (e.g. 50Hz in Europe). Since wind does not flow in constant speed a control has to be applied. There is possibility in reducing the rotor speed (when the wind flows faster than is desired) by changing the pitch angle to reduce lifting force coefficient (coefficient C_L). This process could take a while and therefore there has to be other methods how to prevent change in output frequency. Electrical methods such as change in feeding voltage (synchronous machines) or change in slip (asynchronous machines) are methods which help in stabilizing the frequency very fast and reliable.

Windmills which are not directly connected to the grid rectify produced current and then convert it to AC with desired parameters by inverter which is then connected to the grid. Other possibility is to store energy in batteries (produced current is rectified and then connected to the battery). When the battery reaches its capacity there has to be an option of dissipation of produced current for example to resistors (which can be possibly used for heating etc.). Batteries can then be used as a source of DC or inverted to AC and there is again possibility of connection to the grid.

Since storage of energy is very expensive and in larger scale is impossible, batteries are used only for small stand-alone turbines. Larger wind turbines are either connected to the grid directly or their power produced is adjusted along grid properties before.

Conclusion

General background of wind turbines (HAWT) and their components were given in this chapter. There are also other properties and factors of wind turbines which were not mentioned in this text such as mathematical model of tower of the wind turbine etc.; these can be found in [1] or [15]. Information from the text above will be used in the very next part where the model of the wind turbine will be made.

3. Part II: Antarctica project

The main goal of this paper is development of wind and windmill models. In previous text necessary background was given and with use of data of wind measurements and data from windmill manufacturer mathematical models mentioned above will be created.

3.1 Wind modeling

In this chapter several models of wind will be introduced and one of them will be applied on wind speeds data from a site in the Czech Republic. Wind direction is being omitted from modeling since windmills are able to turn the rotor into the direction of wind (yaw), therefore the only needed parameter of wind is its speed.

Statistical method

Wind modeling method presented in this chapter is presented in [1] - few modifications will be done to improve the results. Results of wind modeling based upon measured data will be shown and discussed in Part III and also in interactive demo (see Appendix D – Demo).

This method is based upon statistical analysis of measured data from given site using information about wind patterns and behavior presented in Part I. Unfortunately data from Antarctica were not available at the time therefore data from a site in the Czech Republic will be analyzed and wind model for that site will be developed. Data available are mean wind speeds of measurements with period 1 second over a period of 10 minutes for one year. For each mean value there is a maximum and minimum wind speed measured in this 10 minutes period.

First of all annual pattern has to be found by finding average wind speed $\overline{U}_m$ for each month along with its standard deviation σ_m:

$$\overline{U}_m = \frac{1}{N_m}\sum_{i=1}^{N_m} U_{m,i} \tag{3-1}$$

$$\sigma_m = \sqrt{\frac{1}{N_m}\sum_{i=1}^{N_m}(U_{m,i} - \bar{x}_m)^2} \tag{3-2}$$

Where N_m is number of measurements in month *m*; $U_{m,i}$ stands for measured value in month *m* (10 minutes average values) and $\bar{x}_m$ is median of month *m*. Median was used instead of mean to minimize the impact of erroneous measurements which usually have extreme values (mean would be affected significantly more).

From the equations above it is possible to model an annual pattern based upon measurements of one year. Next step is to develop diurnal pattern by computing mean wind speed $\overline{U}_{m,h}$ and standard deviation $\sigma_{m,h}$ for each hour *h* in month *m*.

$$\overline{U}_{m,h} = \frac{1}{N_{m,h}}\sum_{i=1}^{N_{m,h}} U_{m,h,i} \tag{3-3}$$

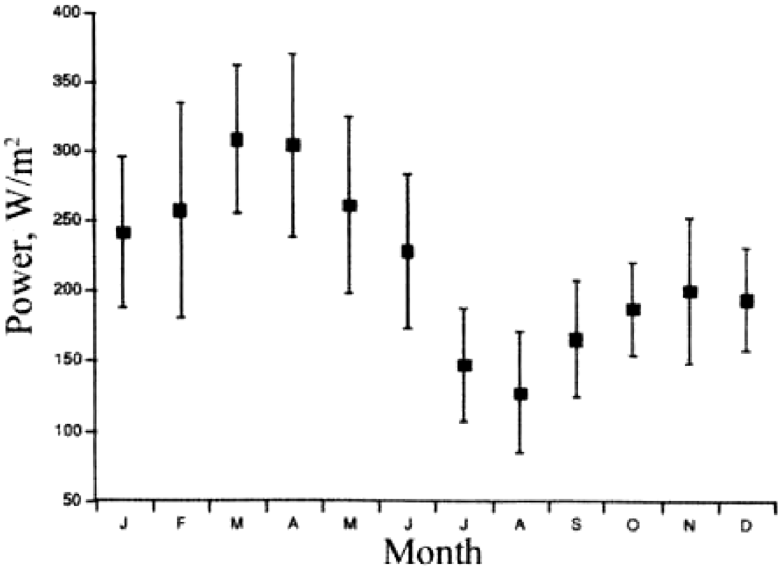
$$\sigma_{m,h} = \sqrt{\frac{1}{N_{m,h}}\sum_{i=1}^{N_{m,h}}(U_{m,h,i} - \bar{x}_{m,h})^2} \tag{3-4}$$

Where *N* is a number of measured mean wind speeds $U_{m,h,i}$ in hour *h* and month *m*; $\bar{x}_{m,h}$ stands for median of measured values *N* in month *m* and hour *h*. Calculated values for annual pattern are in the table below (calculations of diurnal pattern are in Appendix C – Results).

	Average Wind speed [m·s^{-1}]	Standard deviation [-]
January	7,99	4,40
February	6,70	3,62
March	6,98	3,48
April	5,88	2,75
May	6,83	3,06
June	5,00	2,32
July	4,96	2,40
August	6,79	3,27
September	7,77	3,29
October	8,23	3,40
November	10,19	4,17
December	8,22	4,26

Table 3-1: Average wind speeds and standard deviations for each month.

For correct mean wind speed generation lower and upper bounds has to be stated because if only statistics were followed there would be a certain possibility for example to have mean wind speed 0, which is definitely not possible. Bounds would be stated reasonably if measurements for multiple years were available. In the figure below is shown possible bounds selection.



3-1: Example of upper and lower bounds for mean wind speeds of each month.

So far pattern of whole year can be generated. Following text describes how to generate particular wind speeds along generated hour mean values. Rayleigh distribution introduced in Part I was chosen as distribution of wind speeds for its simplicity. Auto-correlation described also in Part I has to be selected. Unfortunately sufficient data are not available for observing time-correlation phenomenon (measurements with period less than 1 minute would be needed) therefore correlation coefficient has to be made out. Generated wind speeds over a time period of 10000 seconds along gradually changing mean value from 6 to 12 $m \cdot s^{-1}$ are shown on the figure below. Correlation coefficient was chosen to be 0.8 (following wind speed can differ only by 20% of the mean from the value previous). Very same correlation method has to be applied on generating mean hour values.

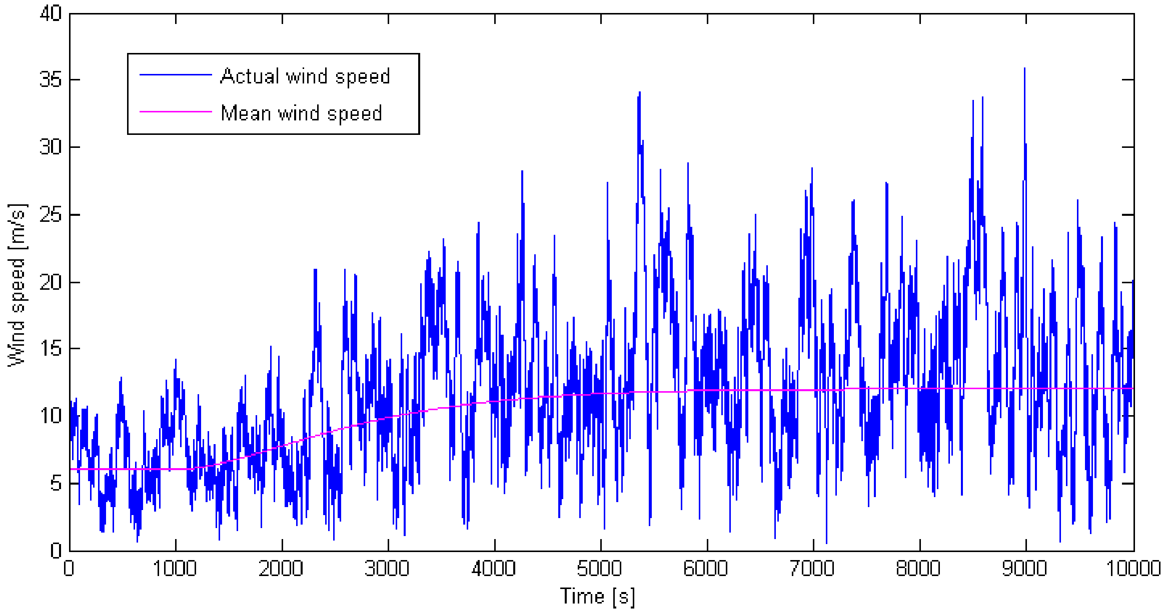


Figure 3-2: Generated wind speed along gradually changing mean with correlation coefficient 0.8.

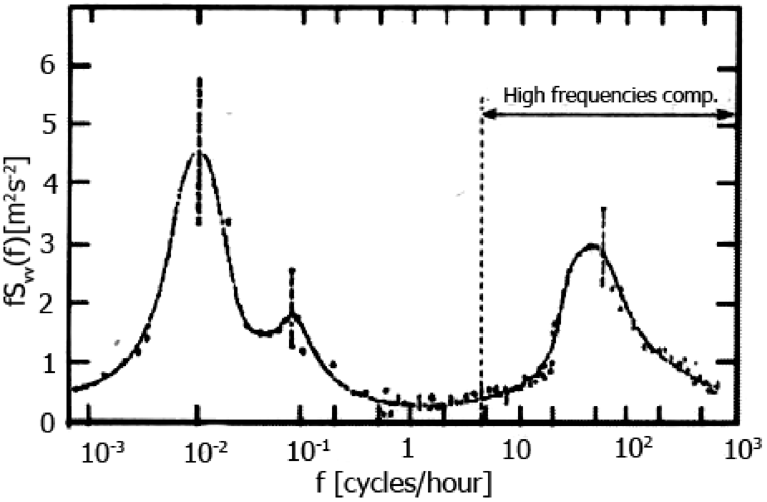
Generated mean values for each hour changes infinitely fast, in other words in 59th minute mean values jumps to mean value of following hour. This unnatural behavior was treated by application of second order filter as it is shown on figure 3-2.

Using statistical method wind model can be generated using simple statistical tools. Several coefficients were made up due to lack of sufficient data. These assumptions and uncertainties does not change model signitificantly and it can be with no doubts use as an input of the model of wind turbine.

Nichita's method: Spectral method

In this text method developed by Cristian Nichita with his colleagues will be briefly presented. For detailed description and wind speed model development see [2]. Nichita's wind speed modeling method is based upon combination of two wind models developed in past. These models are presented below.

For modeling of medium and long-term wind speed variations is used Van der Hoven's spectral model, which is considered to be one of the best large-band wind speed modeling method[26]. Power spectrum of the horizontal wind speed is calculated from 0.0007 to 900 cycles per hour which contains long and medium-term wind speed variations as well as well as turbulence component. Wind modeling is based upon sampling the spectral model illustrated below.



3-3: Van der Hoven's spectral model.

Using only Van der Hoven's spectral model final result would not be sufficiently accurate because turbulence component would be modeled as a stationary process as it can be seen from the figure below. To improve the short-term component of this model von Karman's model is introduced

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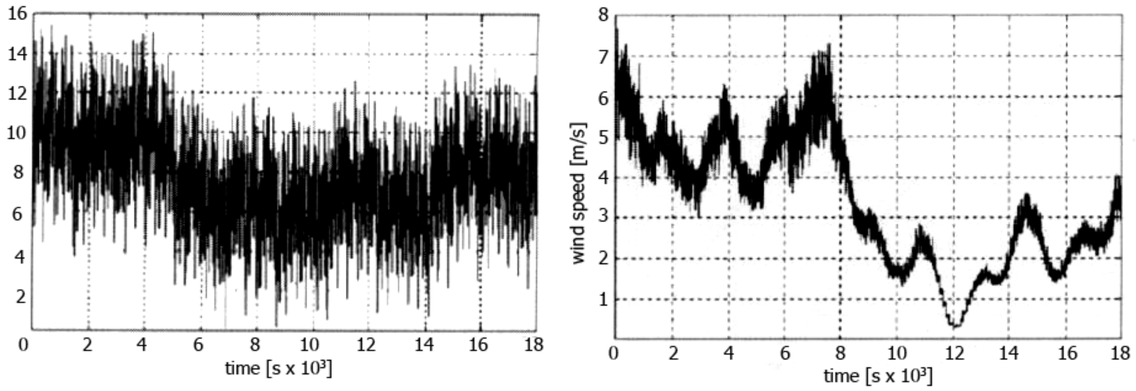
[26] [2] p.253



3-4: a) Wind speeds generated by solely using Van der Hoven's model. b) Wind speed generated along Nichita's method (combination of Van der Hoven's and von Karman's models with adjustable shaping filter).

The turbulence model as a non-stationary process based upon a mean value of wind with specific characteristics (turbulence intensity and turbulence length scale) is given by von Karman's power spectrum. Using von Karman's model only short-term variations can be modeled, however in combination with Van der Hoven's model very good results can be obtained for both long and short-term wind speed variations as can be seen on the figure above.

Wind speeds generated by Nichita's method can be used for experimental testing of control systems strategies since generated wind speed variations are similar to ones of specific site (in case that parameters of von Karman's model were measured experimentally at the site). For short-term component the wind model is similar to the wind speed when measured with an anemometer[27].

Conclusion

There are many other wind modeling methods except these mentioned above. Wind modeling itself as it was presented in this document is useful for getting rough idea about wind conditions respective power production over a long time period. It has to be noted that model developed in this way from data obtained only from one year period does not have sufficient reliability - it is a very rough estimation. Short time scale predictions in terms of hours or days are usually used to predict very precisely the power production of the given windmill which is used to avoid possible shortages by covering them from other energy sources. These predictions are based both on measured data from past and actual or even

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[27] [2] p.525

future information from meteorological stations. Wind model created above using statistical tools will be used in the Part IV. as an input for the windmill model which will be developed in the very next chapter.

3.2 Windmill modeling

Overview

The main goal of this chapter is to develop dynamic model of the wind turbine located in Antarctica (static model is power curve from the manufacturer). In the following text individual components of the windmill will be modeled using background given in Part II and data obtained from the manufacturer of the wind turbine. Wind turbines in Antarctica are connected to batteries where is the produced energy stored, model of batteries and rectifier are left out from this text.

Rotor: angular velocity ω

Rotor of the windmill is composed of three airfoils with profile NACA 4415. Properties of the blades and whole rotor are summarized in the table below.

Number of blades	Rotor radius	Length of blade	Weight of blade	Pitch angle
3	1.675 m	1.450 m	2455 g	2.5 °

Table 3-2: Properties of the rotor.

The simplest possible dynamic model of a windmill is given by the following differential equation.

$$J\frac{d\omega}{dt} = T_T(U,\omega) - T_G(\omega) \qquad (3\text{-}5)$$

On the left hand side is total dynamic moment where J [kg·m^2] is rotor inertia and ω [m·rad^{-1}] is rotor angular velocity. $T_T(U,\omega)$ [N·m] represents torque of the turbine, which is the function of wind speed U [m·s^{-1}] and rotor angular velocity ω. $T_G(\omega)$ [N·m] stands for the load torque of the generator acting on the rotor, which is the function of ω.

First of all moment inertia J of the rotor has to be calculated. Since the shape of the airfoils is complicated few reasonable assumptions have to be made. There are two possibilities how to calculate rotor's inertia. Let's consider the rotor with three blades as a disk of radius R

equal to the radius of the rotor and with mass *m* equal to the mass of a blade[28]. Moment of inertia *J* of rotating disk is given by:

$$J = \frac{1}{2} 3mR^2 \tag{3-6}$$

Or formula for moment of inertia of infinitely thin rod which has to be multiplied by three (moments of inertia of individual blades added together) can be used[29]:

$$J = 3\frac{1}{3}mR^2 \tag{3-7}$$

Actual moment of inertia of the rotor is somewhere between these two values.

General torque is given by:

$$T = \frac{P}{\omega} \tag{3-8}$$

P [W] stands for power.

Substituting equations (2-8) and (2-9) from Part II into the formula (3-8) following is obtained:

$$J\frac{d\omega}{dt} = \frac{1}{2}\rho\pi R^3 \frac{C_p(\mathrm{TSR},\beta)}{TSR} U^2 - T_G(\omega) \tag{3-9}$$

or

$$J\frac{d\omega}{dt} = \frac{1}{2\omega}\rho\pi R^2 C_p(TSR,\beta) U^3 - T_G(\omega) \tag{3-10}$$

Frictional forces are neglected in this part of modeling. These forces will be taken in account later in the text when the load torque of the generator T_G will be calculated (because both are acting against the torque of the rotor T_T).

For correct dynamics it is necessary to know C_p as a function of *TSR* characteristic since β is constant (it can be possibly adjusted manually, default setting is 2.5°). From the manufacturer two characteristics are available: power produced by the windmill as a function

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[28] [10]Development of a Wind Turbine Simulator for Wind Generator Testing

[29] [11], p.274

of wind speed and power produced by the generator as a function of angular velocity of the rotor. Putting these two characteristics together desired C_p versus *TSR* dependency can be found. It has to be noted that in the C_p coefficient will be covered also losses in the generator η_{Gen} and frictional forces, because there is no possibility how to determine these coefficients separately from given data.

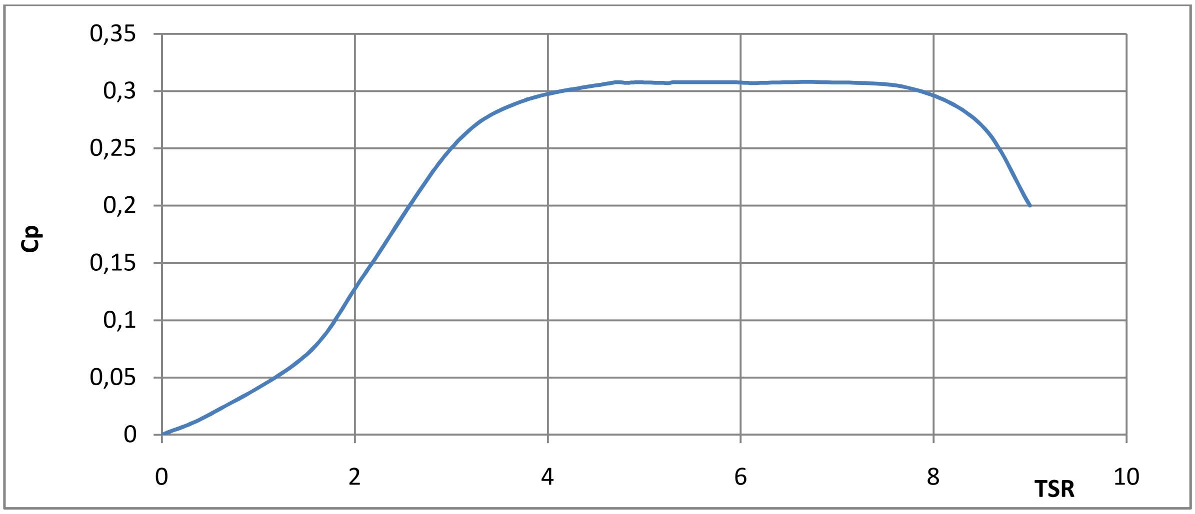

Figure 3-5: Characteristic of C_p as a function of *TSR*.

Characteristic shown in Part II (Figure 2-15b) can be compared with characteristic obtained from manufacturer's data and calculations. It is clear that efficiency of both turbines is very similar – the difference between the curves is caused by losses in the generator which are not included in the power coefficient from the Figure 2-15b.

Since the dynamics of the turbine's rotor are described and gearbox is not present in turbines in Antarctica the very last component of the windmill which has to be modeled is the generator. Model of the generator will be discussed in the next chapter.

Generator: load torque T_G

Generator used at the windmill in Antarctica is 14-pole synchronous machine with permanent magnets. No other information is available about utilized generator. There is only one unknown which has to be determined for completing the mathematic model of the whole turbine and that is characteristic of generator's electromagnetic torque T_G as a function of the angular velocity of the rotor ω. As it was mentioned in the previous chapter efficiency of the generator η_{Gen} is not necessary to be determined because it is already included in the characteristic of power coefficient C_p (figure 3-5). Characteristic of power produced P as a function of rotor's angular velocity ω was used together with power curve to determine the dependence of electromagnetic torque T_G on the rotor's angular velocity ω. Obtained characteristic was compared with T_G vs. ω characteristic of another synchronous generator as it is illustrated below.

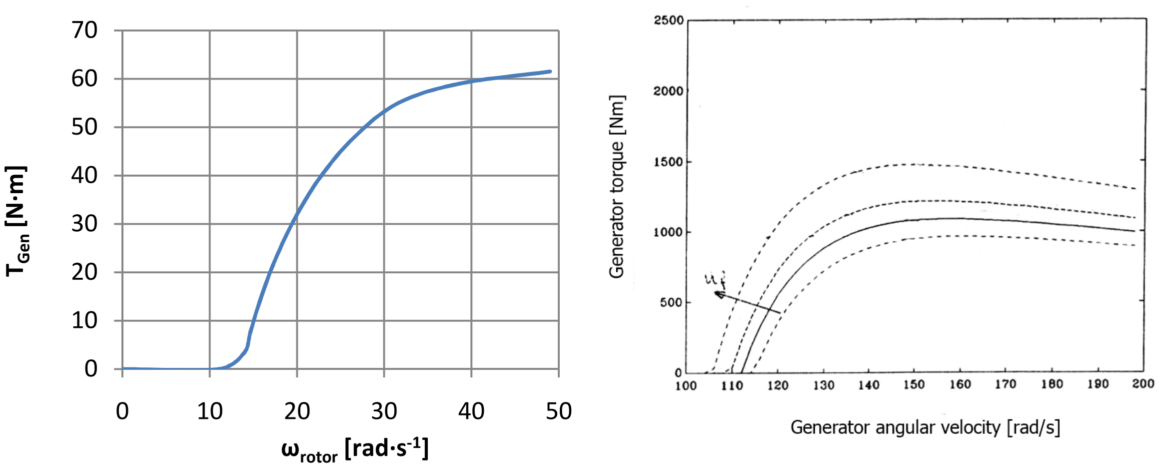

Figure 3-6: a) Generator load torque of windmill in Antarctica derived from data from the manufacturer. b) An example of measured load torque of a synchronous generator[30].

There are also frictional forces acting in the same direction as torque of the generator – against torque of the rotor. These forces are included either in characteristic 3-6a or 3-5; since there is no possibility how to separate them when no measured data or detailed information from the manufacturer about the generator and turbine are available.

In the text above necessary information of forces acting against the rotor and wind torque were introduced. It has to be noted that it is assumed that the load of the generator is constant otherwise model of batteries would have to be developed and generator's toque would have to be adjusted along it.

Conclusion

Complete mathematical model of the wind turbine was developed. Many assumptions and estimations were done, but it should not significantly change turbine's properties and behavior. For better and more precise results more information and data from the manufacturer would be needed. In the very next chapter model will be tested by the model of the wind. Dynamics of the wind turbine are shown on step responses below where wind

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[30] [15], p.23

speed changes from standstill to 5 m·s^{-1} and to 6 m·s^{-1}. Simulink model is present in Appendix C – Results.

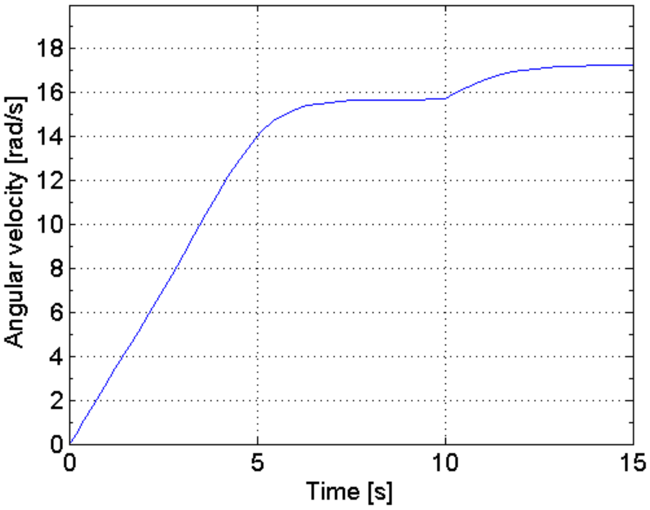

Figure 3-7: Step response of wind turbine (wind speed changes from standstill to 5 m·s^{-1} which is followed by step to 6 m·s^{-1}).

4. Part III: Results

In this chapter model of the wind from 3.1 will be presented and used as an input into the model of the wind turbine created in the previous chapter. Produced power will be discussed and also significance of the wind modeling will be pointed out.

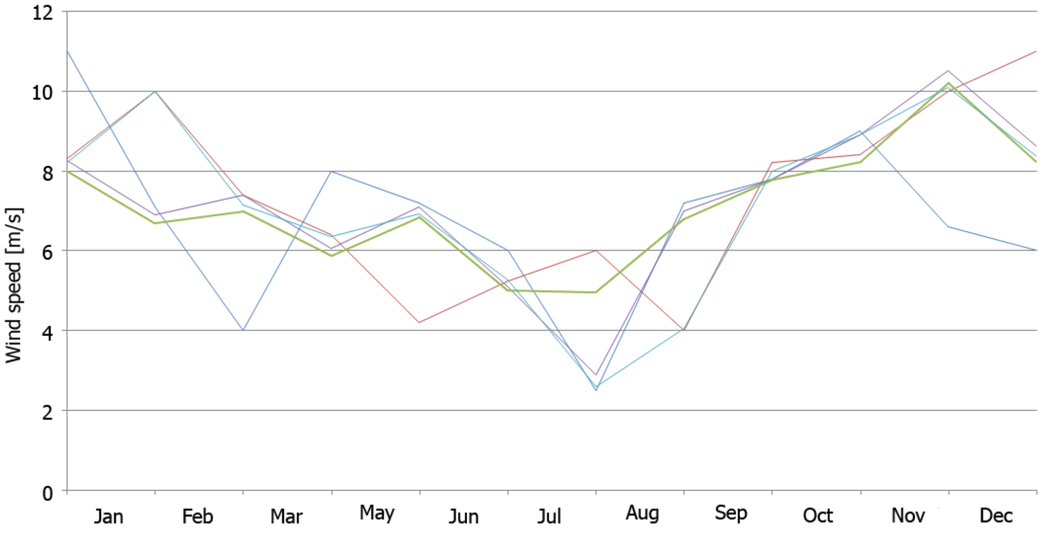

Figure 4-1: Several generated annual patterns along method from 3.1 in comparison with measured data (green line).

In the figure above several annual patterns are generated using statistical method from 3.1 and compared with measured data. Since there were data only from one year period, upper and lower bounds had to be made up. Average month wind speeds are not correlated - there is no information how strong the correlation is (e.g. if windy February with U_m=12 m·s^{-1} can be followed by calm March with U_m = 4 m·s^{-1}). Since there is a large number of measurements for each hour in a month diurnal pattern was generated from them easily with no adjustments as u can see on the figure below. On the other hand there were severe complications with generating particular wind speeds. Correlation was set to value 0.8 for time interval 0.1 second (particular wind speed U can be followed by wind speed in the interval o $U \pm (1-0.8) * U_{mean}$).

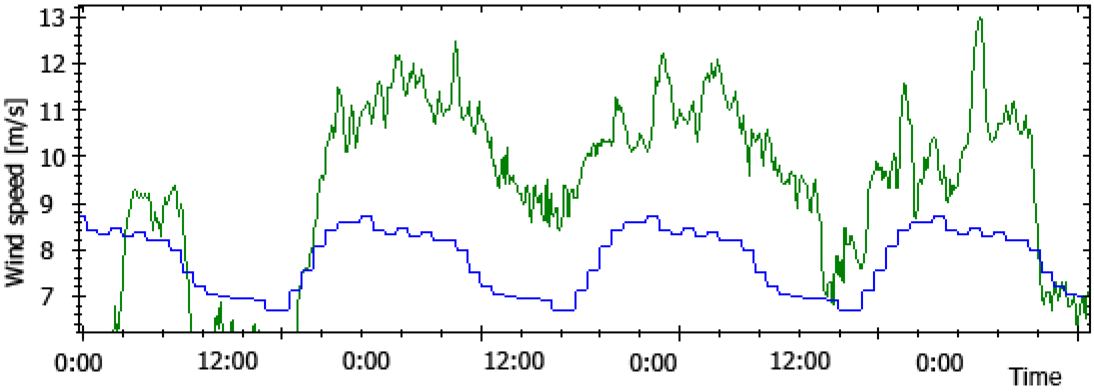

Figure 4-2: Day pattern of wind speeds (green curve: measured data, blue curve: generated wind hour average wind speeds).

Generated one hour period with mean value 4 m·s^{-1} was used (wind speeds were generated with period 1 second) as an input of the model of the windmill. Power production and wind generation are shown on the figure below.

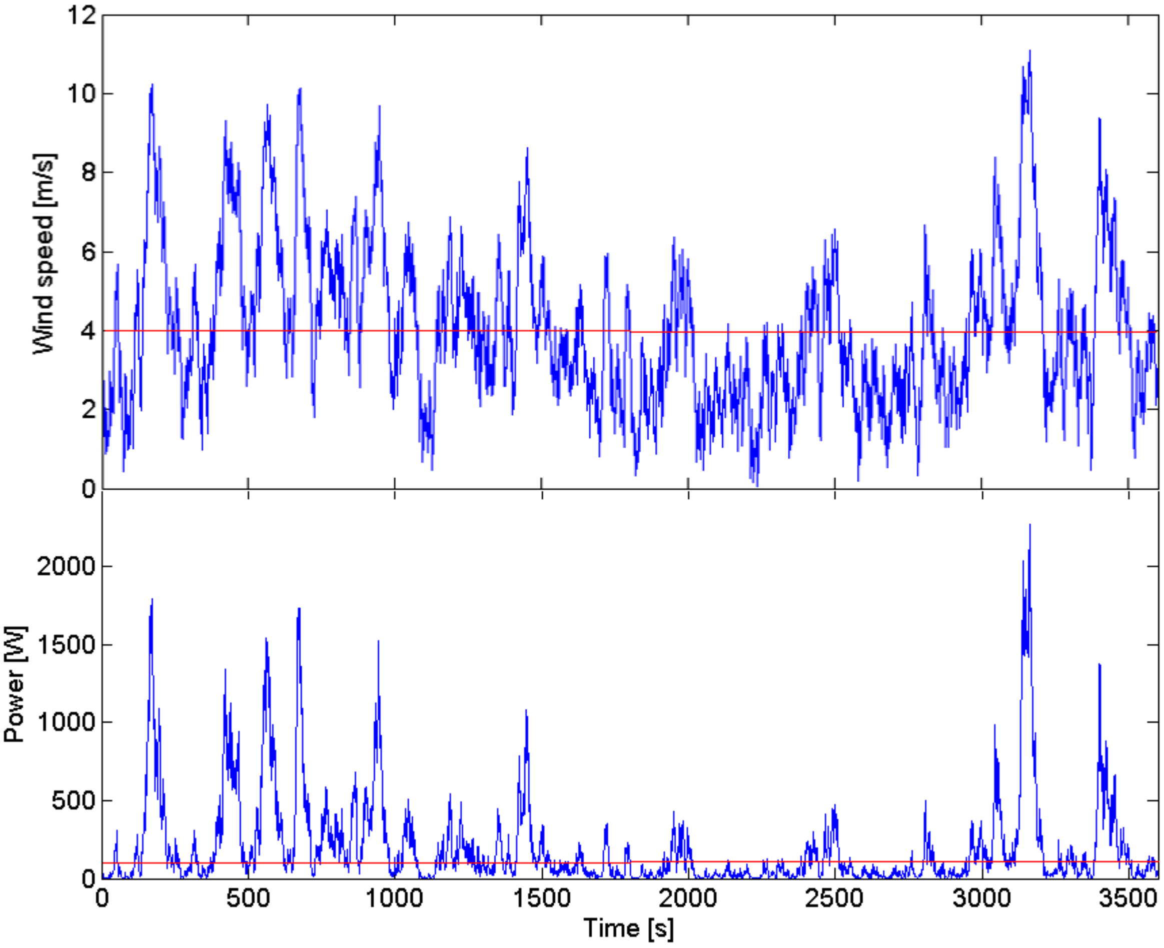

Figure 4-3: Upper graph: Generated wind speeds with period of 1 s (blue curve) with mean wind speed 4 m·s^{-1} (red curve). Lower graph: Power generated by the model of turbine with constant wind speed input of 4 ms-1 (red curve) and with input of generated wind speeds from Figure 4-2 (blue curve).

If only mean wind speed would be used for calculation of the mean power produced in a given time interval an incorrect value would be obtained. In the following calculations the difference will be shown.

$$P_{MeanS} = \frac{1}{2}\rho A U^3 C_p \tag{4-1}$$

$$P_{GenS} = \frac{1}{N}\sum_{i=1}^{N} P_i(U, C_p) \tag{4-2}$$

Power produced when wind is constantly flowing 4 m·s^{-1} is equal to $\boldsymbol{P_{MeanS} = 106\,W}$, on contrary mean power produced by turbine when wind is generated by statistical method and its mean speed is 4 m·s^{-1} is equal to $\boldsymbol{P_{GenS} = 204\,W}$ (several assumptions were made such as mean C_p value was used for this calculation or dynamics of the rotor were excluded, the result should not be affected significantly by them). The difference is caused by the cubic dependence of power on the wind speed. Power P_{GenS} gets much closer to the power production of real wind turbine and is „almost twice as much as we figured out in our naive calculation (4-1)"[31].

5. Conclusion

Throughout this paper model of the wind based upon wind speed data from a site in the Czech Republic was made using statistical method. Due to lack of sufficient data another wind modeling method was only described. Mathematical model of windmill located in Antarctica was developed using given physical background and data obtained from the manufacturer.

Model of the wind does not fully satisfy my personal expectations. Wind speed even for low average values fluctuates too much in comparison with wind models from other documents. This is probably caused by Rayleigh distribution which was chosen for its simplicity – better solution would be Weibull distribution, which can be utilized after data for determining Weibull parameters will be available. Data containing wind speed measurements with period 1 second or less would be needed to improve this model (for example to find out time-correlation coefficient). Overall model of the wind can be used as an input for windmill model to test its dynamics and power production.

Manufacturer of the windmills located in Antarctica was not able to provide me sufficient information about the turbine, however with several reasonable assumptions model was created. Since there are no measured data from the site and I have no access to the turbine itself model dynamics cannot be compared with the real machine. Static behavior corresponds with the given power curve.

Since calculations and all programming used in this paper are in the general form when data from Antarctica will be available model of the wind can be easily computed for this site as well as the model of the windmill can be improved. Practical results and theoretical background presented in this document can be later used as a tool for wind analysis, power prediction or grid stability in the Czech Republic.

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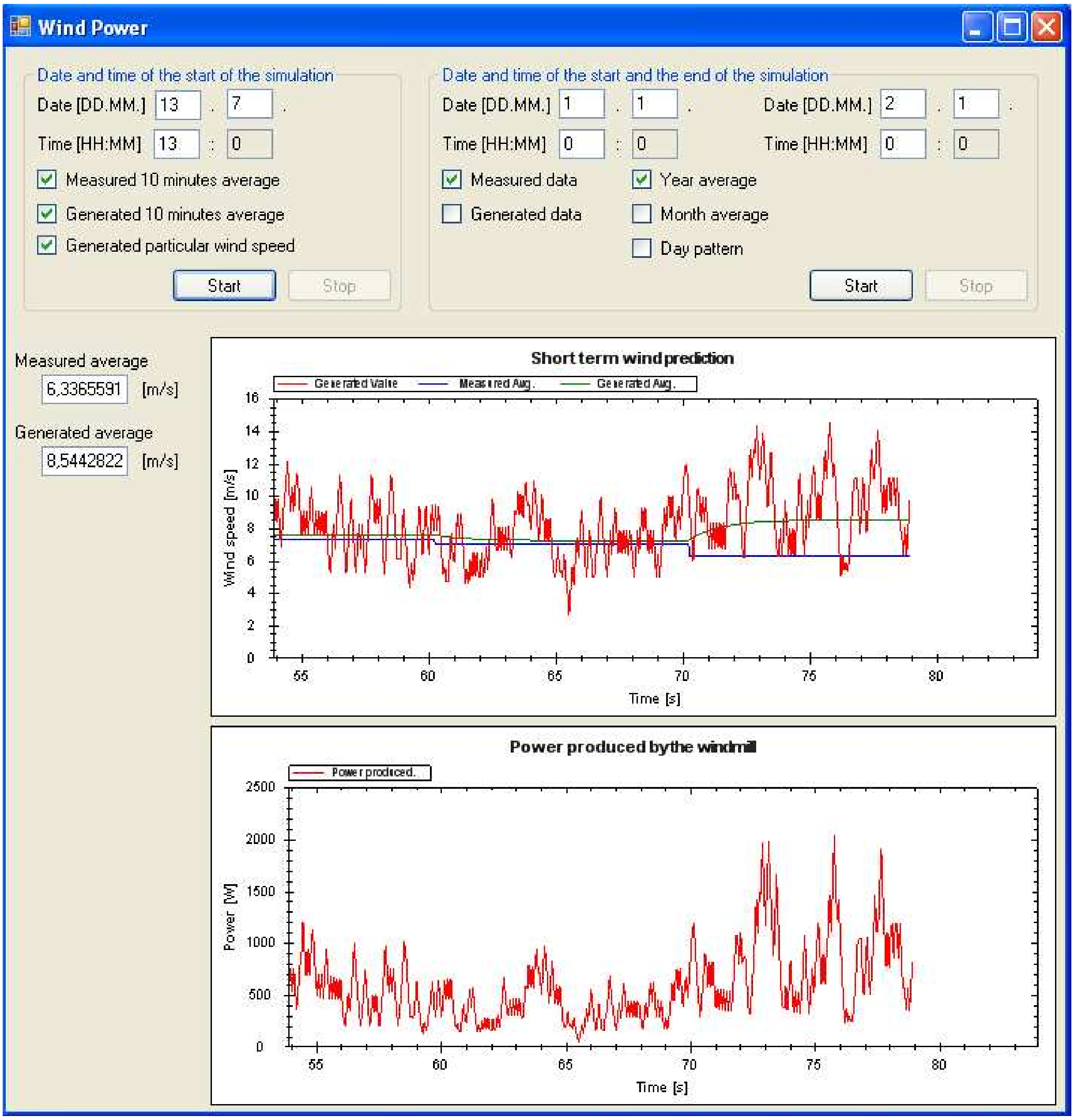
[31] [13], Guided tour>Energy output>Mean power of the wind

Interactive wind generation together with power production is presented in the attached demo. See Appednix D – Demo for more details about the program.



Date and time of the start of the simulation

Date [DD.MM.] 13 . 7 .

Time [HH:MM] 13 : 0

☑ Measured 10 minutes average

☑ Generated 10 minutes average

☑ Generated particular wind speed

Date and time of the start and the end of the simulation

Date [DD.MM.] 1 . 1 .

Time [HH:MM] 0 : 0

Date [DD.MM.] 2 . 1 .

Time [HH:MM] 0 : 0

☑ Measured data

☐ Generated data

☑ Year average

☐ Month average

☐ Day pattern

Measured average 6,3365591 [m/s]

Generated average 8,5442822 [m/s]

Figure 3-1: Screenshot of the demo while wind is being generated (upper graph) and power produced by the wind turbine calculated (lower graph).

6. Sources

Used sources

[1] J.F. Manwell, J.G. McGowan, A.L. Rogers, "Wind Energy Explained", *University of Massachusetts,* 2002.

[2] C. Nichita, D. Luca, B. Dakyo, E. Ceanga, "Large Band Simulation of the Wind Speed for Real Time Wind Turbine Simulators", *IEEE Transactions on Energy Conversion,* Vol. 17, No. 4, p. 523-529 20002.

[3] Ing. Karel Máslo, CSc., "Modelování větrných elektráren pro výpočty dynamické stability", *Obnovitelné zdroje a jejich začleňování do DS, ČEPS,* 2004.

[4] Kira Grogg, "Harvesting the Wind: The Physics of Wind Turbines", *Physics and Astronomy Department, Carleton College*, 2005.

[5] David-Pieter Molenaar, "Cost-effective design and operation of variable speed wind turbines", *Delft University Press,* 2003.

[6] N. A. Cutululis, M. Ciobotaru, E. Ceanga, M. E. Rosu, "Real Time Wind Turbine Simulator Based on Frequency Controlled AC Servomotor", *University "Dunarea de Jos" of Galati, Faculty of Electrical Engineering,* 2002.

[7] G. L. Johnson, "Wind Energy Systems", *Chapter 4—Wind Turbine Power; Wind Turbine Power, Energy, and Torque,* 2001.

[8] K. E. Johnson, L. J. Fingersh, "Methods for Increasing Region 2 Power Capture on a Variable Speed HAWT", *National Renewable Energy Laboratory, Colorado,* 2004.

[9] Anca D. Hansen, Clemens Jauch, Poul Sørensen, Florin Iov, Frede Blaabjerg, "Dynamic wind turbine models in power system simulation tool DIgSILENT", Risø National Laboratory, Roskilde, 2003.

[10] Development of a Wind Turbine Simulator for Wind Generator Testing

[11] D. Halliday, R. Resnick, J. Walker, „Fundamentals of Physics", 1997

[12] http://www.weibull.com, Reliability Engineering Resources for the Reliability Professionals.

[13] http://www.windpower.org, Danish Wind Industry Association.

Other relevant sources

[14] J. Elsner, P. Mixa, P. Prošek, A. Suchánek, Z. Venera, O. Vícha, "The Czech Antarctic Station of Johann Gregor Mendel – from project to realization", 2006

[15] P. Bongers, "A integrated dynamic model of a flexible wind turbine", *University of Technology, Delft*, 1990.

[16] M. Steinbuch, F. Meiring, "Analysis and simulation of a wind turbine with variable speed", *Kema Scientific & Technical Reports 4 (7),* p. 71-78, 1986.

[17] http://colonizeantarctica.blogspot.com, Colonize Antarctica Blog.

[18] http://geography.uoregon.edu/weather, Department of Geography, University of Oregon.

[19] http://www.stloe.most.go.th, Science ant Technology Knowledge Center.

[20] http://www.mathworks.com, Matlab Specialized Plot and Graph Types.

[21] http://www.localsustainability.org.uk

[22] S. Kocman, "Synchronní stroje", *Katedra obecné elektrotechniky FEI VŠB, Ostrava*, 2000

Appendix A – Physics

Betz' Law

The reasonable assumption has to be made before the coefficient Cpmax can be found: average speed of wind through the rotor of turbine is average of speeds through A1 and A3 (see figure 2-10), (v1 + v2)/2.

The mass of air flowing through the turbine in one second is given by:

$$m = \rho A \frac{(v_1 + v_2)}{2} \tag{A-1}$$

Where ρ is air density, A represents swept rotor area.

The power extracted fro the wind is equal to the mass times drop of the wind speed squared after passing the rotor.

$$P = \frac{1}{2} m ({v_1}^2 - {v_2}^2) \tag{A-2}$$

Substituting *m* into this expression from the first equation (A-1) we get power extracted from the wind given by:

$$P = \frac{\rho}{4} A (v_1 + v_2)({v_1}^2 - {v_2}^2) \tag{A-3}$$

Let us compare the energy P_w in the wind flowing through the very same area *A*, without any obstacles such as turbine's rotor blocking it.

$$P_w = \frac{\rho}{2} A {v_1}^3 \tag{A-4}$$

The ratio between power extracted and power trapped in the wind is given by:

$$\frac{P}{P_0} = \frac{1}{2}\left(1 - \frac{v_2}{v_1}\right)^2 \left(1 + \frac{v_2}{v_1}\right) \tag{A-5}$$

After plotting the expression (A-5) we may determine the maximum power which can be extracted from the wind.

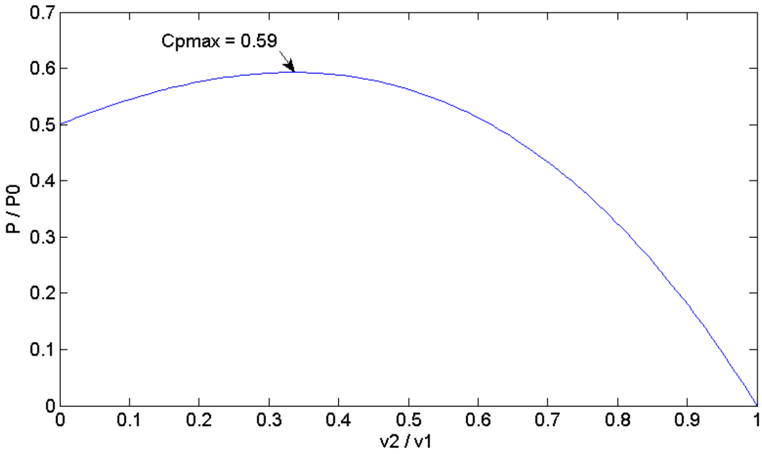


Figure A-1: C_{pmax} is by the value of the maximum of P/P_0 as a function of v_2/v_1.

The maximum of the function at figure A-1 is $C_{pmax}=0.59$ for $v_2/v_1=1/3$. This value is only theoretical, in reality C_{pmax} has lower value because of reasons mentioned in 2.1.

Appendix B – Statistics

Rayleigh distribution

Probability density function of Rayleigh distribution is given by:

$$p(x) = \frac{xe^{\left(\frac{-x^2}{2\sigma^2}\right)}}{\sigma^2} \tag{B-1}$$

Mean is given by:

$$\mu(X) = \sigma\sqrt{\frac{\pi}{2}} \tag{B-2}$$

Putting these two equations together pdf in terms of $\mu(X)$ and x can be represented by:

$$p(U) = \frac{\pi}{2}\left(\frac{U}{\mu^2}\right)e^{\left[\frac{-\pi}{4}\left(\frac{U}{\mu}\right)^2\right]} \tag{B-3}$$

Appendix C – Results

Simulink model

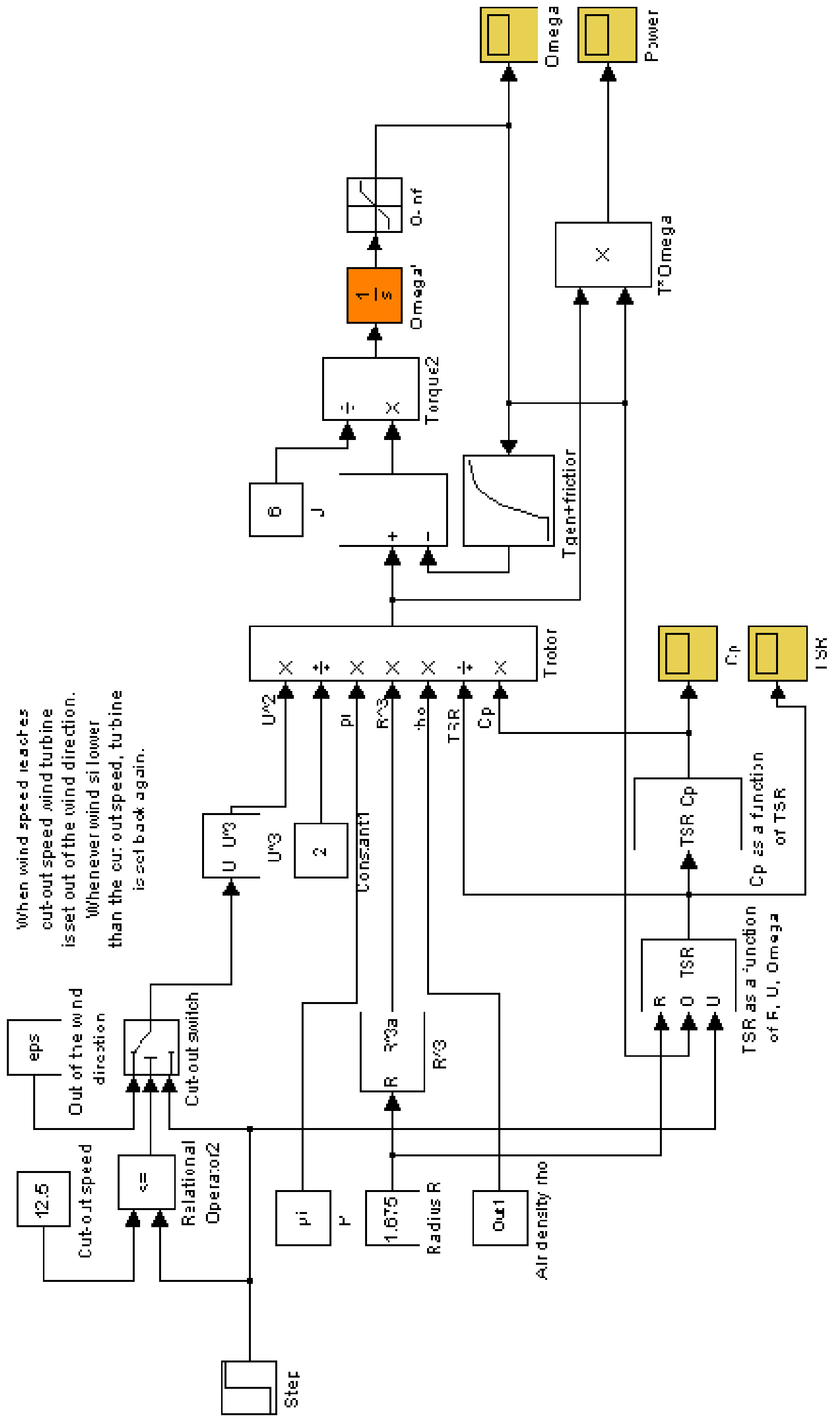


Table of hour means and standard deviations

Hour	$\overline{U}_{Jan}$	σ_{Jan}	$\overline{U}_{Feb}$	σ_{Feb}	$\overline{U}_{Mar}$	σ_{Mar}	$\overline{U}_{Apr}$	σ_{Apr}	$\overline{U}_{May}$	σ_{May}	$\overline{U}_{Jun}$	σ_{Jun}	$\overline{U}_{Jul}$	σ_{Jul}	$\overline{U}_{Aug}$	σ_{Aug}	$\overline{U}_{Sep}$	σ_{Sep}	$\overline{U}_{Oct}$	σ_{Oct}	$\overline{U}_{Nov}$	σ_{Nov}	$\overline{U}_{Dec}$	σ_{Dec}
0:00	8,52	3,65	6,82	2,67	7,33	3,26	6,32	2,80	6,09	2,54	5,73	1,89	5,12	2,26	7,02	2,71	8,73	3,43	8,55	3,83	10,38	3,66	8,57	3,23
1:00	8,78	3,48	6,93	2,92	7,19	3,33	6,31	2,41	6,27	2,47	5,51	2,00	5,13	2,49	7,18	2,81	8,41	3,15	8,39	3,52	10,19	3,59	8,22	3,28
2:00	8,39	3,73	6,75	3,06	7,04	2,90	6,44	2,52	6,58	2,53	5,59	2,03	5,20	2,75	7,30	3,10	8,34	3,01	8,68	3,81	10,37	3,80	8,27	3,32
3:00	8,25	3,45	6,70	3,22	7,15	2,97	6,60	2,09	6,53	2,55	5,39	2,15	5,00	2,76	7,18	3,22	8,47	3,21	8,53	3,36	10,05	3,52	8,14	3,41
4:00	8,05	3,81	6,96	3,09	7,21	2,92	6,32	1,85	6,72	2,88	5,38	2,15	5,23	2,67	7,13	3,17	8,27	3,16	8,79	3,37	9,80	3,63	8,01	3,10
5:00	7,68	3,89	6,95	3,15	7,18	2,86	6,33	2,10	6,74	2,78	5,16	2,13	5,15	2,66	7,10	3,04	8,40	3,30	9,00	3,47	10,23	3,90	8,07	3,22
6:00	7,59	3,67	6,72	3,06	7,27	3,00	5,86	2,32	6,71	2,83	5,07	2,65	5,07	2,57	6,91	3,00	8,19	3,47	9,01	3,58	10,16	4,05	8,22	3,39
7:00	7,13	3,80	6,84	3,04	6,99	3,18	5,36	2,20	6,71	3,02	4,81	2,48	4,50	2,28	6,34	3,41	8,20	3,16	8,81	3,63	9,70	3,91	8,32	3,57
8:00	7,47	4,20	6,98	3,10	7,02	3,36	5,27	2,20	6,91	2,79	4,59	2,22	4,70	2,25	6,44	3,40	7,99	3,11	8,64	3,58	10,15	3,95	8,52	4,12
9:00	7,99	4,41	6,88	3,04	6,94	3,53	5,33	2,48	7,19	2,68	4,58	2,17	4,71	2,28	6,76	3,50	7,53	3,21	8,23	3,61	10,27	3,99	8,82	4,13
10:00	8,39	4,30	6,88	3,83	6,85	3,65	5,44	2,89	7,47	2,85	4,65	2,12	4,71	2,13	6,91	3,22	7,22	3,06	7,94	3,51	10,00	4,03	8,80	4,13
11:00	7,92	3,97	6,17	3,43	6,90	3,39	5,73	2,91	7,61	3,07	4,77	1,93	5,05	2,07	7,03	3,04	7,05	2,78	7,94	3,56	9,83	4,27	8,69	4,31
12:00	7,84	4,00	6,33	3,39	6,97	3,36	5,79	2,77	7,94	3,12	4,96	2,26	4,88	2,02	6,80	2,88	6,98	2,68	7,68	3,42	9,60	4,31	8,37	3,96
13:00	7,65	3,78	6,43	3,55	6,83	3,46	5,50	2,35	7,67	3,23	5,09	2,18	4,97	1,70	6,75	2,79	6,97	2,45	7,33	3,32	9,79	4,30	8,49	4,17
14:00	7,30	3,75	6,90	3,98	6,42	3,43	6,09	2,63	7,53	3,35	5,09	2,27	5,03	1,73	6,68	2,75	6,97	2,51	7,02	3,54	9,72	4,15	8,55	4,38
15:00	7,25	3,53	6,75	3,51	6,48	3,33	6,04	2,61	6,99	2,91	4,81	2,10	5,01	1,65	6,34	2,60	6,91	2,57	7,23	3,56	9,96	4,27	8,08	4,21
16:00	7,69	4,07	6,28	3,18	6,74	3,01	5,87	2,39	6,65	2,59	4,77	2,18	5,09	2,20	6,21	2,50	6,71	2,48	7,23	3,48	10,21	4,28	8,24	4,15
17:00	8,14	4,26	6,45	3,14	6,84	2,96	5,98	2,63	6,51	2,88	5,04	2,25	5,10	2,44	6,26	2,39	6,71	2,36	7,65	3,29	10,30	4,32	8,51	3,97
18:00	8,20	4,23	6,85	3,42	6,56	2,89	5,85	3,00	6,35	2,53	4,64	2,20	5,15	2,35	6,35	2,51	7,11	2,64	8,14	3,16	10,68	3,77	8,46	3,64
19:00	8,32	4,49	6,86	3,49	7,00	2,94	5,74	2,88	6,54	2,60	4,65	2,20	4,58	1,75	6,83	2,62	7,56	2,88	8,23	2,98	10,67	3,60	8,57	3,36
20:00	8,26	3,93	6,63	3,38	6,91	2,89	5,49	2,59	6,46	2,68	4,68	2,44	4,40	1,55	6,85	2,69	8,06	3,08	8,11	3,12	10,56	3,49	8,59	3,40
21:00	8,27	3,88	6,55	3,03	7,11	3,11	5,75	2,50	6,70	2,32	4,68	2,27	4,79	2,01	6,83	3,05	8,42	3,20	8,49	2,93	10,63	3,66	8,45	3,54
22:00	8,19	3,82	6,54	2,75	7,27	2,99	5,74	2,63	6,68	2,27	5,00	2,06	5,13	1,95	6,96	3,00	8,59	3,47	8,85	3,27	10,77	3,80	8,19	3,00
23:00	8,50	3,75	6,61	2,63	7,45	3,33	5,94	2,64	6,31	2,35	5,36	1,98	5,42	1,88	6,82	2,85	8,61	3,62	9,06	3,52	10,61	3,65	8,54	3,15

Appendix D – Demo

Demo was developed for modeling of long-term (months/days) and short-term (minutes/seconds) wind speed variations using method presented in 3.1.

Data needed for calculations which are being loaded by the program are in two text files: times.txt and avgs.txt. These files have to be located at C:\DATA\ ; times are in format "d.m.yyyy h:mm" (one time for each line). Each time corresponds with average wind speed in avgs.txt. Average wind speeds for each 10 minutes are in format "10.3" [m/s]. Software reads all data at startup therefore these data can be switched for other measurements from different site to model wind speeds for this given site.

For short-term modeling it is necessary to chose starting time from when wind will be generated. This can be easily done by filling up text boxed in the upper left corner and pressing START button. Measured and/or generated mean hour wind speeds can be plotted in upper graph together with particular wind speeds generated each 100 ms. In the lower graph the power production of windmill (modeled in 3.2) is demonstrated.

To initialize long-term wind speed model start and stop times have to be selected. This can be done by filling up text boxes in the right upper corner. After pressing START button curves are plotted along check box selection.

Graphs can be zoomed by selecting desired area; by pressing right mouse button characteristic can be zoomed back.

It has to be noted, that loading the demo takes a while (around 1-2 minutes), because more than 100 000 values has to be read and utilized for computation of constants.