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Your ref:	Our ref:	Attended by:	Bratislava
	01/2023	**prof. Ing. Michal Kvasnica, Dr.sc.**	**Nov 6th, 2023**

Review of PhD Thesis of Vít Cibulka

The proposed PhD thesis of Mr. Cibulka is concerned with analysis and control of nonlinear dynamical systems using the Koopman and Sum-of-Squares (SOS) approaches. The thesis spans 109 pages and is divided into two principal parts. The first topic is thoroughly discussed in Chapters 2 to 6. There, the candidate first introduces the reader to theory of the Koopman operator in Chapter 2. The exposition is technical (and technically correct), yet accessible even to non-experts. The reviewer appreciates clarity and thoroughness of the introduction to the Koopman operator and finite truncations thereof, and acknowledges the importance of supplementing the theoretical formulations with discussion about the practical aspects, such as complexity of the resulting Koopman MPC formulations, along with hints of selecting the appropriate controller structure.

Chapter 3 reports state-of-the-art methods for synthesizing the finite truncations of the Koopman operator, reviewing all major and popular approaches. The chapter also clearly identifies deficiencies of the current methods, namely, the need for an a-priori selection of the dictionary of lifting functions. This topic is of imminent importance, because the proper choice of such a dictionary is paramount, especially for control practitioners, to the success of the Koopman approach.

Candidate's main theoretical contributions in the Koopman part are summarized in Chapter 4. Specifically, it reports novel results that allow to directly synthesize the lifting functions (along with the matrices of the Koopman dynamics) straight from measured (or simulated) trajectories of the dynamical system. The idea is very sound and is backed up by solid theoretical development and rigorous proofs. The reviewer appreciates clarity of the explanation where the candidate's technical results are accompanied with motivating examples. Also appreciated is the thoroughness of the technical discussion that covers all practical aspects of the implementation, ranging from preparation of trajectories, through initialization of variables, up to handling practical control setups like those including slew rate constraints. In the reviewer's opinion, the material of Chapter 4 is novel and significantly contributes to state of the art by providing a theoretically sound, yet very practical approach to synthesis of finite-dimensional approximations of the Koopman operator. It needs to be highlighted that the proposed approach directly supports systems with discontinuous inputs, which is a non-trivial task.

Chapter 5 highlights the applicability of the proposed approach on various motivating examples, clearly showing the benefits of the candidate's approach over existing methods in terms of approximation accuracy and general applicability. Moreover, the chapter discusses the application of

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various solvers to solve the resulting optimal control problems based on Koopman models. The first part is then summed up in Chapter 6, which reports the highlights.

The second part is devoted to Sum-of-Squares methods to the computation of regions of attraction for nonlinear dynamical system. There, Chapter 7 first introduces the concept of SOS in a very didactical manner. The reviewer appreciates that this chapter can serve as a study material for both undergraduate as well as graduate students in relevant courses (e.g., on optimization, optimal control, etc.).

Candidate's main results are then reported in Chapters 8 and 9. There, the thesis proposes a novel way of finding the outer approximation of the region of attraction (ROA) by splitting the problem both in time as well as in space. By doing so, the level of conservatism can be significantly reduced, as demonstrated by the examples reported in Section 8.4. The reviewer highlights that the theoretical and technical development presented in Chapter 8 is technically sound and is backed up by rigorous proofs. Especially important is the proof guaranteed outer approximation (cf. Theorem 1).

The novel results of Chapter 8 are further extended in Chapter 9 that provides a proof of differentiability of the resulting SDP relaxations. Additionally, the chapter also further optimizes the splitting method that yield significant improvement in terms of the objective value while allowing to use just simple polynomials. The second part of the thesis is then summed up in Chapter 10, which provides the main highlights.

After a thorough reading of the thesis, this reviewer has come to a firm conclusion that the PhD thesis of Mr. Cibulka fulfills all conditions for it to be accepted for defense. The thesis undoubtedly contains novel material that contributes to scientific knowledge in the field of analysis and control of dynamical systems. The candidate, through his thesis, and his other scientific publications too, has demonstrated his ability and readiness to conduct independent scientific research. Therefore, this reviewer provides a **pass** assessment of the PhD thesis of Mr. Cibulka.

Questions:

- Can the candidate provide a motivating case where both the Koopman and the SOS methods would be used simultaneously?
- Please discuss scalability of the methods of Chapter 5 with respect to growing state and input dimension.
- One of the challenges of applying Koopman-based methods is the generation of trajectories. How do you propose to proceed when dealing with open-loop unstable systems?
- Even if the system to be modeled using the Koopman approach is open-loop stable, generation of trajectories could still be challenging in practice. Often, the system to be modeled cannot be (easily) decoupled from its underlying low-level controller, which is often based on logic rules. Does the candidate see a way of processing such closed-loop trajectories and then taking the low-level controller's dynamics "out of the equation"?

- Does the candidate see a way of applying the Koopman framework not to the dynamics of the system itself, but rather to derive a closed-form model of a cost function that can only be measured, but otherwise behaves as a black box? This would open up the methodology to handle many real-world problems where the cost function represents, e.g., reactions of markets to decisions taken by the Koopman MPC.
- In the SOS ROA approach, is it possible to get an a-priori bound on the approximation error?

Sincerely,

prof. Ing. Michal Kvasnica, Dr.sc.
Full professor of automation