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MECHATRONIK  
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Ihre Zeichen, Ihre Nachricht vom

Unsere Zeichen

, Bearbeiter

Report on thesis of D. Wagner

Report thesis Wagner

A. Schirrer

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### Report on the dissertation thesis of Daniel Wagner

Dear madam or sir,

Following your kind invitation I have reviewed the dissertation thesis "Measures and LMIs for V&V of Adaptive Control" submitted by Ing. Daniel Robert Wagner, MSc. in the field "Control Engineering and Robotics" at the Faculty of Electrical Engineering, Czech Technical University in Prague.

The thesis shows a specific approach to solve the verification and validation (V&V) problem of polynomial aircraft models with nonlinear, adaptive control laws. This approach utilizes occupation measures and structured LMI relaxations (called the moment sums-of-squares (SOS) or Lasserre hierarchy) to produce numerical certificates on stability or performance, and it also directly provides critical (pathological) system trajectories resulting from the associated primal optimization problem formulation.

The thesis is structured in a logical manner and succeeds to provide the complex matter both in its theoretical background and in relevant aerospace test cases. This report addresses the main aspects of the thesis' evaluation as structured below.

**Relevance** The thesis topic is highly relevant to the current research needs in the scientific control and aerospace community. Verification and validation of the safety, reliability and performance of control systems are essential particularly in the aerospace domain. Multiple systematic approaches have been formulated to tackle the difficult problems of finding, identifying and assessing critical closed-loop validation scenarios that occur with low probability in which Monte-Carlo-based methods are unsuitable. The reviewed thesis takes on a structured approach to directly find pathological trajectories in control settings with polynomial plant dynamics, adaptive control laws, or unmodeled flexible system dynamics.

This is done by formulating occupation measures and hierarchical linear matrix inequalities (LMI) relaxations, directly leading to numerical certificates verifying stability and/or performance, respectively an expression of critical system trajectories.

**Fulfillment of main objectives** The thesis aims to provide a set of methods and tools to perform the validation and verification task on polynomial (aircraft) system models with nonlinear feedback. This is achieved via the proposed framework based on occupation measures and LMI relaxations. Numerical certificates are produced as a result, and it is found that they can guarantee boundedness of all trajectories and finite-time convergence, which is a qualitative difference over traditional Lyapunov-based validation methods. The V&V-problem is first posed as polynomial dynamical optimization problem. Its equivalent infinite-dimensional linear programming (LP) problem of measures is expressed. This latter problem can then be relaxed and solved using truncated moment LMI sequences with off-the-shelf software and semi-definite programming (SDP) solvers. Additionally, techniques to exploit sparsity of systems of ordinary differential equations (ODEs) are employed, so that more complex V&V tasks become tractable. This is demonstrated for an elaborate aircraft model with model-reference adaptive control (MRAC) laws, which lead to additional states and hence higher complexity in its validation task. Finally, the V&V-methodology has been extended to specifically address unmodeled and/or uncertain system dynamics, such as uncertain flexible-modes coupling or uncertain actuator dynamics. Numerical certificates that consider such bounded uncertainties can thus be obtained, which is demonstrated in two application use cases. The objectives of the work have thus been fulfilled entirely and demonstrated at relevant aerospace test problems.

**Appropriateness of methods** The methods chosen to address the thesis' contents and goals are appropriate and effective. Employing occupation measures and the Lasserre hierarchy approach to transform complex, polynomial optimization problems into LMI / LP problems allows to directly solve for the pathological scenarios and trajectories and allows to formulate stability / performance certificates numerically. This approach is well-suited for V&V-tasks despite its numerical effort and possible tractability issues (which are specifically addressed in the thesis), complementing the widely-applied Monte-Carlo-based empirical V&V-methods in the field. Optimization and simulation methods, as well as elaborate Lyapunov-based stability assessment complete the range of employed methods to develop and demonstrate the thesis' results. Numerical simulations demonstrate the V&V-results found by the developed framework and show comparisons with results obtained by classical Monte-Carlo studies.

**Main results and contributions** This thesis employed moment SOS hierarchies for V&V of aerospace problems with adaptive control laws such as MRAC. Numerical certificates for the polynomial aircraft model with adaptive feedback can be done with the proposed V&V-framework.

The main contributions of this thesis are:

**advanced algorithms and a V&V-framework** for polynomial aircraft models with nonlinear control laws such as MRAC. This provides an alternative to traditional V&V-methods that are insufficient for adaptive control, such as Monte-Carlo, or only provide infinite time convergence analysis of non-piecewise polynomial systems. Both, longitudinal and lateral F-16 plants with MRAC have been validated, and the produced

numerical certificates guarantee boundedness of all trajectories and convergence in finite time.

**exploiting sparsity of ODEs** to significantly reduce the computational effort associated with the considered V&V problems, so that more complex tasks remain tractable. A complex problem for the lateral F-16 plant with piecewise disturbances and MRAC feedback is successfully solved.

**incorporating bounded uncertain parameters** via their explicit representation in the space of measures to allow to study relevant robustness problems in the V&V context. It is shown that the MRAC can tolerate at least sufficiently small unmodeled dynamics within the studied F-16 control settings (including uncertain coupling to aeroelastic modes and uncertain actuator dynamics).

Five chapters organize the technical content of the thesis:

**Chapter 2** outlines the theoretical preliminaries including the setting of polynomial dynamical optimization, occupation measures, piecewise polynomial dynamical optimization, parsimony, and a standard MRAC formulation.

**Chapter 3** outlines the V&V problem of a polynomial longitudinal dynamics of an F-16 fighter aircraft controlled with MRAC. The problem is first shown as a polynomial dynamical optimization problem, followed by the transformation steps needed to convert it into a feasible moment SOS relaxations problem. Three cases with piecewise disturbances are discussed.

**Chapter 4** presents a new method to exploiting sparsity for Ordinary Differential Equations (ODEs) in the V&V context with which the MRAC reference trajectory is approximated. For V&V of a lateral F-16 polynomial model with MRAC with piecewise disturbances, the resulting large number of states can effectively be tackled by exploiting ODE sparsity.

**Chapter 5** treats the V&V problem of MRAC in the presence of unmodeled flexible dynamics. Uncertain parameters are represented in an explicit way in the space of occupation measures. An F-16 linear/polynomial model and MRAC is considered with uncertain parameters coupling its longitudinal dynamics to its aeroelastic modes. It can be certified that sufficiently small parameters exist for which no unsafe trajectories arise.

**Chapter 6** shows the handling of uncertain higher order actuator dynamics containing uncertain parameters. Analogous to the robustness results in Chapter 5, it is verified that the MRAC can tolerate limited actuator uncertainties.

The thesis' scientific contents have been published in two peer-reviewed papers by the author at two renowned control conferences (chapters 3, 4) and submitted (but not yet accepted) at the International Journal of Control (chapter 5) and at another peer-reviewed conference (chapter 6). The technical content of these contributions is sound and properly presented, but more details, background and tutorial contribution would be desirable to be more easily accessible for readers of the general control community. A broader backing in terms of the number of accepted, peer-reviewed publications would be desirable (and the reviewer is not aware of the precise corresponding formal PhD requirements regarding publication requirements), but the content and its compilation in this thesis seems correct

and of acceptable quality for a PhD dissertation thesis. The arrangement of the topics successfully spans a logical bridge from the basic realization of the method (chapter 3) over various relevant extensions (chapters 4–6) with convincing results.

**Importance for further development of science** This thesis adds a novel and distinct set of methods and tools to the field of validation and verification analysis of polynomial systems with nonlinear control. The results are applicable of course in aerospace use cases, in which the thesis contents have been brought forward. However, they also seem to be beneficial in other application areas, particularly in control use cases in which adaptive control is or could be employed, but in which it may need more stringent guarantees of stability, safety, and performance to be applied on a wider scale.

The main concern of the methodology is certainly its limitation in terms of tractability and the strong dependency of the complexity of the solution construction on the formulation of the system dynamics. Improving applicability and efficiency of such relaxation hierarchies is an active field of research (and has been addressed in this thesis, too) with the prospect to unlock a wide application potential of LMI relaxation-based analysis methods. The thesis provides an outlook that also lists possible further fields of study, including state estimation and observer-based MRAC, or a comparison with existing region-of-attraction V&V-formulations.

It is noted here that the thesis does provide a compact introduction to the complex matter of occupation measures and LMI relaxations, but does not serve as a tutorial to these methods – a general control engineering’s audience will have to resort to external sources to grasp the entirety of these approaches. The thesis does, however, provide applied examples to sufficiently outline the capabilities of the proposed V&V-framework.

**Creative scientific work** This thesis represents a creative scientific contribution by providing novel, challenging realizations of the V&V-approach via LMI relaxations for polynomial control systems and nonlinear feedback. A logical sequence of extensions is presented throughout the thesis in a convincing way. Illustrative aerospace test problems highlight the methods and their potential in application generally well, however more focus on general audiences would have unlocked significant tutorial value which the thesis currently lacks. Nevertheless, the V&V-methodology is successfully explored and extended in this work, serving as a foundation and inspiration for future work.

**The author of the thesis proved to have an ability to perform research and to achieve scientific results. I do recommend the thesis for presentation with the aim of receiving a Ph.D. degree.**

Yours sincerely,

Alexander Schirrer