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Title: Scheduling with uncertain processing times given by empirical distributions

Overview of the work

The thesis deals with several scheduling problems with uncertain processing times. The main goal of the is to develop optimization algorithms for scheduling problems with uncertain processing times. The thesis is divided into sections, where each is devoted to a different problem and a different approach to represent the uncertainty in job processing times. **Each of the problems studied in Sections 2-5 is published in a different high-quality journal, which is already a clear signal about the high quality of the work**. After the Introduction section, the following sections are included:

- Section 2. In this section the student focus on a scheduling problem on identical parallel machines. The objective is to assign jobs to machine and to provide a job processing order on each of the machines such that the worst-case value of the expected total completion time is minimized. In this section, the(uncertain) processing times are expressed by a random vector subject to an ambiguous probability distribution.

  Among the results that the student achieved in this part are the following:

  - He reformulated the problem as a minimization of a linear function plus a robust term in the sense of l_2 norm.
  - He investigated the effect of the form of the robust term on the computational
    complexity of the problem, and thereby improve the best-known upper bound on the complexity for the problem with independent jobs.

- He extended the method to the case of dependent job processing times and provided some insight about the cause of hardness.

A paper was accepted to publication in EJOR based on the results obtained in this section.

- **Section 3.** In this section the student studied a single machine-scheduling problem where jobs have a criticality index. The uncertainty in job processing time is expressed by a set of possible processing times, and it is unknown (a priori) which processing time will be realized during the schedule. If the processing of a job is prolonged, it may lead to the rejection of other jobs. The objective is to construct a feasible schedule (where the exact definition of a feasible solution is clearly defined in the section) that minimizes the makespan. The problem known to be strongly NP-hard even if there are only two possible criticality indices. The authors:
  - Present a 3/2-approximation algorithm to solve the case where there are two possible criticality indices.
  - Present a novel MILP formulation for the problem.
  - Developed a Brach and Price algorithm to solve the problem.
  - Showed that the optimal schedule can be presented in a form of a tree.

  A paper was accepted to publication in EJOR based on the results obtained in this section. In fact, I was among the reviewers of this paper, and was really impressed by the result obtained. Here is a sentence from my evaluation "I really like the paper. It is well-written, and the authors did a very nice work both in analyzing the problem and in testing the algorithms. Overall, I found that the paper is good enough to allow publication in EJOR."
- **Section 4**: The student introduced the concept of job replication as a tool to increase the probability that a job will be executed in a mixed-criticality scheduling problem. He proposed an algorithm to solve the problem of computing execution probability for a job. The algorithm utilizes the theoretical

framework of Bayesian networks. **A paper was accepted to publication in AOR based on the results obtained in this section.**

- **Section 5:** In this section the student focused on an identical parallel machine scheduling problem, where the processing time of each job is a stochastic variable that follows a normal distribution function. The objective is to construct a schedule that *maximizes* the probability that all jobs will be finished before a common due date.

  The main contributions the student obtained in this section are:

  - He presented a list scheduling algorithm that provide a lower bound on the solution value which is better than the best known lower bound in the literature.
  - He presented a new upper bound on the objective based on the concept of aggregated machines.
  - He presented a branch-and-price decomposition scheme for solving the problem.
  - He proved that the pricing problem is strongly NP-hard.
  - He provided an exact algorithm for the special case of two machines**.**

  A paper was accepted to publication in EJOR based on the results obtained in this section.

Answer to the Questions

Question 1: To what extent the subject of the thesis is relevant to the current needs of the scientific community.

Answer: Scheduling problems are usually analyzed under the assumption that job processing times are known in advance. However, in many cases this is not the case and there is uncertainty regarding this parameter. The main aim of the thesis is to present several methods to model uncertainty in job processing times and to solve/analyze the

corresponding scheduling problem. The subject is very important as there is a lack in good methods to schedule jobs under uncertainty in job processing times.

Question 2: To what extent the main objectives of the work have been fulfilled.

Answer: I believe that the student made significant contribution to the field of scheduling with uncertainty. He provided new ideas to tackle problems (e.g., present the idea of using job replication in mixed-criticality problems, to increase its likelihood to be executed), and applied new methods to analyze the problems he tackled (e.g., applying methods from machine learning). The presentation is excellent, the analysis is correct (as far as I can tell), and the results are impressive and have been published in excellent journals. Therefore, I find that the main objectives have been satisfied.

Question 3: To what extent the methods used in the thesis are appropriate.

Answer: The student uses a large set of methods from different fields, which is quite impressive. It includes complexity analysis of problems, statistical methods, algorithmic design, approximation algorithms, among many other tools. The thesis includes both theoretical analysis of the problems and practical examination of the algorithms. I found the variety of methods used in this thesis very impressive.

Question 4: What the main results and contributions of the work are.

Answer: The results are highlight above (see my overview of the thesis).

Question 5: to what extent the work is important for the development of science.

Answer: The field of scheduling under uncertainty has many more challenges and I believe that problems, methods, and idea developed in the thesis can be used in many other problems in the field of operations research.

Question 6: whether the thesis satisfies the conditions of a creative scientific work.

Answer: See my conclusion below.

To conclude:

The student study very important problems with a wide range of tools from different fields of knowledge (computational complexity, algorithmic design, statistics, graph theory and more). The presentation is excellent, and the results are highly regarded by the academic community (four different papers have been extracted from the thesis and have been published in a high-quality journals).

The author of the thesis proved to have an ability to perform research and to achieve scientific results. I do recommend the thesis for presentation with the aim of receiving a Ph.D. degree.

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