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Assessment of PhD Thesis

Name of the PhD Student: Petr Augusta

Title: ***Spatially Invariant Systems: Modelling, Analysis and Control via Polynomial Approach***

Name of the examination member: Tomáš Vyhlídal

Thesis objectives, outline and general remarks

The Thesis, which is written in monographic style, consists of eight chapters including introduction and conclusions. The thesis is written very carefully and in very good English. The overall number of pages is 86, the number of references in Bibliography is 75. Besides, the author provides a list of publications where he is the main author or a co-author. The list consists of three journal papers (two published and one in review) and seven conference papers. Four of these papers can be found on the ISI Web of Science. There can be found 6 citations to these papers on ISI too. Already this aspect is very supportive for positive evaluation of the research results of the PhD candidate.

The first, introductory chapter provides definition of the concepts applied in control of the spatially distributed systems. Then, the objectives of the thesis are defined as:

To develop practical methodology for control design for spatially distributed systems via polynomial techniques, in particular to

Objective 1 - give a method to model a spatially invariant system by transfer function
Objective 2 - establish necessary and sufficient condition for stability and develop a simple method to check whether the system is stable or not
Objective 3 - find calculus to design a stabilising controller and a controller optimal in the sense of minimizing the classical quadratic criterion

In the remaining part of the introductory chapter the outline of the thesis, original results and contributions and author's publications related to the thesis are provided.

The second chapter, which is written on two and half pages, is the state of the art. Although it provides the very necessary background to the development of the given research field, in my opinion, it is too brief. The related works are described in words only (except the only equation in the chapter) and the descriptions are too general as rule. The number of references mentioned in the chapter is adequate. However, the references are often grouped and accompanied by very general remarks only.

The third chapter is devoted to the modelling and discretization issues of the spatially distributed systems. First, the most common types of the distributed parameter models are defined; i.e. elliptic, parabolic and hyperbolic PDEs. Besides, two examples of distributed parameter systems are provided. One of these examples – a model of heat conduction in a rod equipped with spatially distributed sensors and heaters – is consequently used as a demonstrative example throughout the thesis. Next, the very basic explicit and implicit schemes for discretizing the systems in both time and space are mentioned and briefly commented. As the second case, the discretization scheme is applied to the spatial dimension only, while the time dimension is left continuous. For the purpose of the thesis, the first order explicit method is used for discretising the model in both the time and spatial coordinates. For this discretization scheme, the stability analysis is performed for one dimensional heat equation, resulting in a stability condition determined by the ratio between the time and spatial steps. Finally, after mentioning briefly the role of the initial and boundary conditions, the simulation of the case study example is performed. Surprisingly, the error analysis by discretization is neither performed nor even mentioned.

The Chapter four deals with definition of a transfer function for describing the spatially invariant systems. For the simplification reasons, the systems are considered infinite in the spatial domain. The system description starts from the input-output convolution integral form for continuous time model and convolution sum for discrete time model. In both cases, the spatial dimension is supposed to be discretised. Thus, the inner infinite sums appear in the convolutions. Utilising the joint Laplace and Z transform for the continuous time model and sequence of two Z transforms for discrete time model, the transfer functions are defined. The transfer functions are in form of a ratio of two polynomials, one sided in s (z) and two sided in w (the operator of discrete spatial domain). Following the systems-over-rings concept, the 2D polynomials are written as polynomials in s (z) with the coefficients as functions of w. For both continuous and discrete time domain, the transfer functions of the case study systems are derived. By the results presented in this chapter the first objective of the thesis is fulfilled.

One of the main results of the thesis - BIBO stability analysis – is derived in Chapter five. Utilizing the Shanks theorem, the stability conditions are provided in Theorem 5.2 and Corollary 5.3 for discrete time models and in Theorem 5.7 and Corollary 5.8 for continuous time models. The condition states that the system is stable iff all s (z) roots of the transfer function denominator are located in the stability region (left half plane/outside the unit circle) for all values of w lying on the unit circle. The application of the given stability criterion precludes the case when the numerator and denominator polynomials have a common factor. The methodology derived for practical evaluation of the stability criterion is based on transforming the problem into checking the positivity of two-sided matrix polynomial. In order to determine stability, defined Schur-Cohn (for discrete system) / Hermite-Fujiwara (for continuous system) matrix needs to be positive-definite for all values of w on the unit circle (Lemmas 5.4 and 5.9). Using semidefinite programming formulation and results of cited works, the problem can be formulated as LMI. Two examples are included to demonstrate the method for a discrete time model. In the first example, it is shown that for a first order system in z the problem can be solved via sweeping the parameter w over the unit circle. The second example demonstrates the application of the stability check of a second order system based on solving LMI. The solution is accompanied by the SeDuMi/Yalmip code. In this chapter, the second Objective of the thesis has been successfully fulfilled. However, I would appreciate more detailed analysis of the BIBO stability for this kind of systems (e.g. even the meaning of abbreviation of BIBO is not mentioned, it is not explained why the unit circle check for w is sufficient, the common factor stability dependence might be discussed in more detail).

The sixth chapter, which deals with polynomial approach to control, provides the main results of the thesis – control design techniques for the spatially distributed systems. First, the positive polynomial approach is used to stabilize both discrete and continuous time systems. It is shown that if the stability analysis derived in the previous chapter is applied to a characteristic polynomial of a closed loop system, it can be used successfully for the synthesis of the controller (Theorem 6.2 and Lemma 6.3 for discrete time system/Lemma 6.9 for continuous time system). However, it applies if and only if the order of the system in time dimension is equal to one. Besides, the general case of higher order system is addressed for a discrete time system. The critical issue is that the stability condition for higher order system is no longer convex. A possible solution is to linearize the condition. In this way, the stability can truly be analysed. However, the utilization of the method in control design is not that straightforward as it is in the first order case. Four examples are included which very nicely demonstrate the results derived. Next, the procedure for LQG – optimal controller is designed. The method, which is proposed in Theorem 6.13 and practically explained in Algorithm 6.14, is motivated by the classical LQG design for 1-D case, which is generalised to 2-D case under consideration. A comprehensive proof of the Theorem is included. One very detailed example demonstrates the application of the designed procedure. LQG controller is designed with the help of MuPAD and polmat Matlab utilities. The code generated for the problem solution is included as a part of the example. The following section on H_2 controller design starts with the Theorem outlining the Youla-Kučera parameterization for multidimensional case. Then the H_2 controller is designed in a more or less standard way. The design procedure is summarized in Algorithm 6.17 and one application example is included. The last control design method proposed in the thesis is the dead-beat control. Analogously as in the H_2 controller design, the methodology is based on the extension of known design techniques for 1-D case. The designed method for dead beat control is presented as Algorithm 6.19 and demonstrated in an application example (here, a figure with manipulated variable might be included next to the other figures). To sum up, there are five control design methods presented in the sixth chapter. All of them are original and technically correct. Besides, the designed methods are well demonstrated on a series of application examples. Taking into consideration these results, the third objective of the thesis is fulfilled.

In the seventh chapter, tools for manipulating the multivariate Diophantine equations which were used in chapter 6 are outlined. First, the conditions on solvability of the equations are provided and next, the problems of spectral factorization of the multivariate polynomials are handled. In chapter eight, two alternative LQG control design approaches are tested and compared with LQG method designed in chapter 6. First a system under consideration is approximated by a high order state space model and the Matlab command lqg is used for the design. Even though the control results are very similar to those obtained in LQG design in chapter 6, the design and computation time in control are high. It is due to the high dimensionality of the models which is transferred to the high size of the control matrices. Secondly, a method based on solving the Riccati equation with polynomial matrices, which was introduced in eighties of the previous century, is applied. The matrices of the derived continuous time model are dependent on the complex variable *w*. It is shown that the method results in irrational optimal control, which cannot be solved directly but needs to be approximated first. The results obtained are with shorter settling time on the one hand, but with larger needed power of control variable when compared with the LQG designed in the thesis. In the conclusion section of this chapter the author highlights the main aspects of all the three methods. However, no quantitative evaluation is performed. In the last concluding chapter, the author first outlines the content of the thesis and summarizes the main results. The main contributions of the thesis are identified as follows

1. Multidimensional BIBO stability analysis
2. Positive polynomial approach to stabilization, for both the discrete and continuous time
3. LQG control design for spatially distributed systems

These results are in agreement with the objective of the thesis. The author then outlines his publication results related to the thesis and provides short comments on future work.

Major comments

The thesis presents high quality and original research results from the area of description, stability analysis and control design of spatially distributed systems in both discrete and continuous time domains. The quality of the presented research is supported by the publication results of the author. The list of publications where the author is the main author or a co-author is remarkable. The fact that some of the results have been published in the top journals and the top (IEEE and IFAC) conferences, is the best evidence of the high quality of the presented research.

As regards the research results presented in the thesis, I have no critical comments. On the other hand, in my opinion, the thesis could have been better structured. Besides, despite of the relatively broad scope of the thesis (it covers the system description, stability analysis and several control design methods), it is written in a rather brief form. The main comments regarding these aspects of the thesis are the following:

1. The list of symbols and abbreviations is missing.
2. Both the introductory and the state of the art chapters of the thesis are too brief. They do not provide sufficient background of the research in the area. Besides, the objectives of the thesis should be determined based on the proper state of the art analysis. Therefore, the objectives should be stated after the state of the art chapter and not before, as it is in the thesis.
3. The chapter on the spatially distributed systems is very brief too. Besides, only first order algorithms for the discretization of PDEs are outlined. Even though the discretization is rather marginal topic of the thesis, it might be explained on more advanced level. Besides, no error analysis of the discretization schemes is performed. In the case of solving the continuous time models, the parameters of the solvers should be mentioned.
4. In Chapter 4, the joint Laplace and Z (or Z - Z) transform should be introduced in more detail before it is used for deriving the transfer function of the system (e.g. convergence features should be mentioned).
5. The aspects of multidimensional BIBO stability should be explained in more detail in Chapter 5. Example 5.1 does not help very much in explaining the effect of nonessential singularity of the second kind. A more detailed background on the Shanks theorem should be provided.
6. In my opinion, Chapter 7 should rather be included as a subsection of Chapter 6, as it presents the known results needed and used in Chapter 6.
7. Quantitative comparison of the methods should be included in Chapter 8.

Minor comments

p. 5, references P4 and P8 – instead preprints, proceedings should be mentioned; p. 11 – K instead of deg. C should be used; p. 18 – Fig. 3.8 – time axis label is missing; p. 19, 21 – cab → can; p. 26 – $H(2\pi)=0$ too; p. 39 – throw → through; p. 52 – must → must be; p. cepstral → spectral.

Questions to be answered during the defence

1. Based on the definition of the system, the actuators and sensors are considered to be placed at the regular discrete positions of the spatial dimension. As it is presented in the thesis, the distance of the actuators is determined by the discretization scheme, which might be technically restrictive. Can the approach designed in the thesis be extended to the case when the actuators are placed in larger distances, i.e. there will be some nodes without control action? In this case, models with time delay might be a proper tool for modeling and control design.
2. In the demonstration example of LQG method (Example 6.13), after the closed loop response to initial condition predominantly settles down in the time axis, it seems that there appears a sort of oscillatory behaviour along the spatial dimension. Could you show how this effect develops further on in time? Can the suppression of these oscillations be included in the control method, e.g. by modifying the quadratic criterion?

3. In the control of time delay systems, a finite order feedback from the state variables $u(t)=-Kx(t)$ (note that K is static gain matrix of relatively low dimension) can be used to stabilize the system, even though the spectrum of the system is infinite [Zítek, (1997), Michiels, et al (2002), Vyhlídal (2003), Vanbiervliet, et al (2008)]. In my opinion, this concept of controlling the dominant modes of the system only (while the remaining modes are checked for stability), might be applied to spatially distributed systems too. In your opinion/experience, are similar approaches applicable (or already used) in the theory of spatially distributed systems?

Overall recommendation

To sum up, the research results presented in the thesis are original and of high quality. Even though I have raised some critical remarks mainly regarding the structure of the thesis, the overall quality of the thesis is high. As also all the objectives of the thesis are fulfilled, I have arrived at the conclusion to **recommend the thesis for defence**.

In Prague, February 24, 2011

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