Static priority scheduling

Michal Sojka

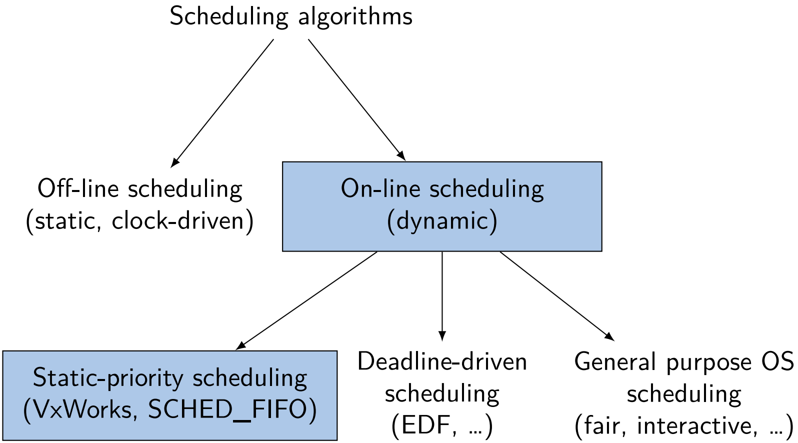
Czech Technical University in Prague,
FEE and CIIRC

December 1, 2021

Some slides are derived from lectures by Steve Goddard and James H. Anderson

Classification of scheduling algorithms

(used in real-time systems)



Outline

Outline

Static priority scheduling

Fixed-priority scheduling

- All jobs of a single task have the same (static, fixed) priority
- We will assume that tasks are indexed in decreasing priority order, i.e. τ_i has higher priority than τ_k if $i < k$.
- We will assume that no two tasks have the same priority.

Notation

- p_i denotes the priority of τ_i.
- $\mathrm{hp}(\tau_i)$ denotes the subset of tasks with higher priority than τ_i.

Basic questions

- How to assign task priorities?
- How to verify that all deadlines are met (schedulability)?

Outline

Rate-Monotonic scheduling

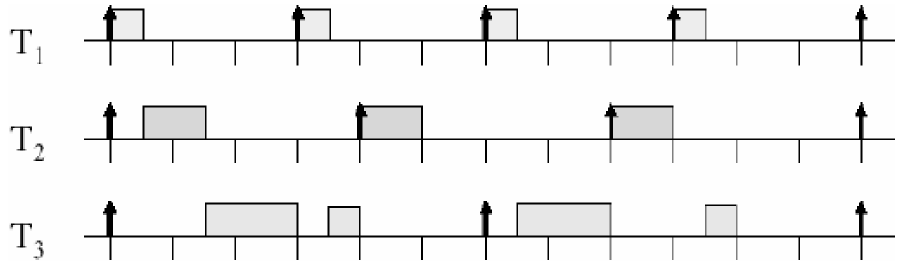
CZ: Rozvrhování podle frekvence (Liu, Layland)

Rate-Monotonic priority assignment

The less period T_i the higher priority p_i.
For every two tasks τ_i and τ_j: $T_i < T_j \Rightarrow p_i > p_j$.

Example (RM schedule)

Three tasks (T, C): $\tau_1 = (3, 0.5)$, $\tau_2 = (4, 1)$ a $\tau_3 = 6, 2$.



Rate-Monotonic priority assignment

Example

Task	Period	Priority
τ_1	25	0
τ_2	60	2
τ_3	42	1
τ_4	105	4
τ_5	75	3

Deadline-Monotonic scheduling

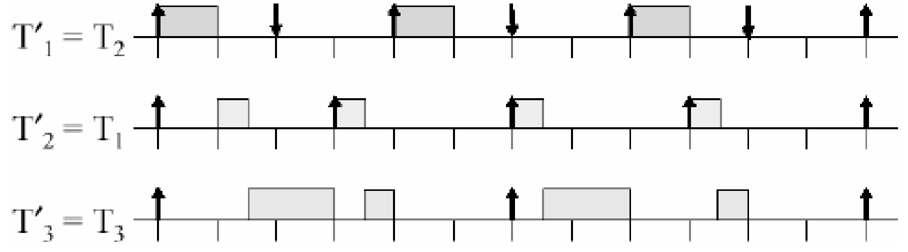
Rozvrhování podle termínu dokončení (Leung, Whitehead)

Deadline-Monotonic priority assignment

The earlier deadline D_i, the higher priority p_i.
Pro each two tasks τ_i a τ_j: $D_i < D_j \Rightarrow p_i > p_j$.

Example (DM schedule)

Let's change the RM example by tightening deadline of $\tau_2 = (T, C, D)$:
$\tau_1 = (3, 0.5)$, $\tau_2 = (4, 1, 2)$ a $\tau_3 = 6, 2$.

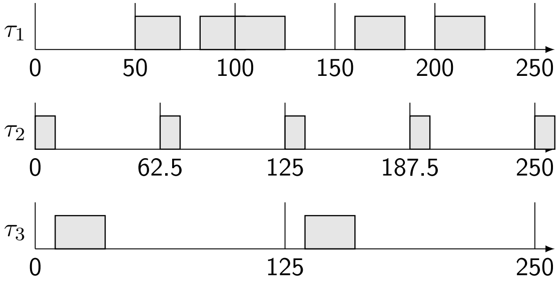


RM vs. DM

- The periodic tasks given by:

task	r_i	T_i	C_i	D_i
τ_1	50	50	25	100
τ_2	0	62.5	10	20
τ_3	0	125	25	50

- DM schedule:

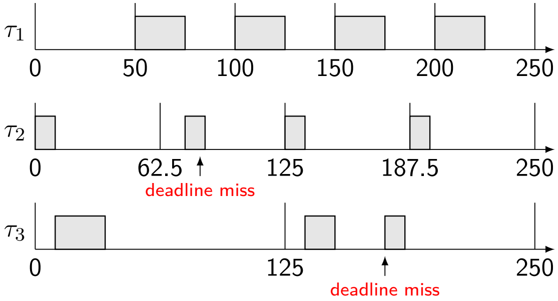


RM vs. DM

- The periodic tasks given by:

task	r_i	T_i	C_i	D_i
τ_1	50	50	25	100
τ_2	0	62.5	10	20
τ_3	0	125	25	50

- RM schedule:



- DM fails $\Rightarrow$ RM fails. DM can produce feasible schedule when RM fails.

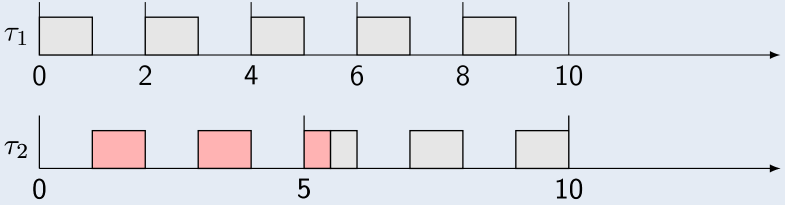
RM and DM optimality

Theorem

Neither RM nor DM is optimal.

Proof.

Consider $\tau_1 = (2, 1)$ a $\tau_2 = (5, 2.5)$. Total system utilization is 1, so the system is schedulable (see the EDF lecture).
However, under RM or DM, a deadline will be missed, regardless of how we choose to (statically) prioritize τ_1 and τ_2.



Simply periodic systems

RM algorithm is optimal when the periodic tasks in the system are simply periodic and the deadlines of the tasks are no less than their respective periods.

Definition

A system of periodic tasks is simply periodic[1] if for every pair of tasks τ_i and τ_k in the system where $T_i < T_k$, T_k is an integer multiple of T_i.

Theorem

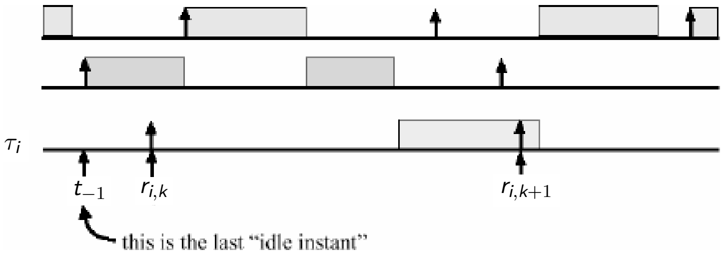
A system $\mathcal{T}$ of simply periodic, independent, preemptable tasks, whose relative deadlines are at least their periods, is schedulable on one processor according to the RM algorithm if and only if its total utilization is at most one.

In practice, people often use periods: 1 ms, 10 ms, 100 ms and 1 s.

[1] CZ: jednoduše periodický

Proof

- We wish to show: $U \leq 1 \Rightarrow \mathcal{T}$ is schedulable.
- We prove the contrapositive: $\mathcal{T}$ is not scheulable $\Rightarrow U > 1$.
- Assume $\mathcal{T}$ is not schedulable. Let $J_{i,k}$ be the first job to miss its deadline.



- **Note:** We suppose that tasks are in phase (not shown in the figure above) and processor never idles before $J_{i,k}$ missed its deadline.

  Because the system is simply periodic, $\dfrac{r_{i,k+1} - t_{-1}}{T_j}$ is integer.

Proof (cont.)

Because $J_{i,k}$ missed its deadline, the demand placed on the processor in $[t_{-1}, r_{i,k+1})$ by jobs of tasks $\tau_1, \ldots, \tau_i$ is grater than the available processor time in $[t_{-1}, r_{i,k+1}]$. Thus:

$$
\begin{aligned}
r_{i,k+1} - t_{-1} &= \text{available processor time in } [t_{-1}, r_{i,k+1}] < \\
&< \text{demand placed on processor in } [t_{-1}, r_{i,k+1}] \text{ by jobs } \tau_1, \ldots, \tau_i = \\
&= \sum_{j=1}^{i} (\text{the number of jobs of } \tau_j \text{ released in } [t_{-1}, r_{i,k+1}]) \cdot C_j \leq \\
&\leq \sum_{j=1}^{i} \frac{r_{i,k+1} - t_{-1}}{T_j} \cdot C_j
\end{aligned}
$$

Proof (cont.)

This we have

$$r_{i,k+1} - t_{-1} < \sum_{j=1}^{i} \frac{r_{i,k+1} - t_{-1}}{T_j} \cdot C_j$$

Canceling $r_{i,k+1} - t_{-1}$ yields

$$1 < \sum_{j=1}^{i} \frac{C_j}{T_j},$$

i.e.

$$1 < U_i < U,$$

This completes the proof.

Optimality among fixed-priority algorithms

Theorem

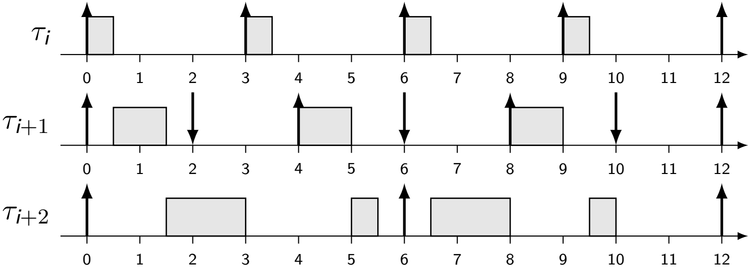
A system $\mathcal{T}$ of independent, preemptable, periodic, synchronous tasks that have relative deadlines at most their respective periods can be feasibly scheduled on one processor according to the DM algorithm whenever it can be feasibly scheduled according to any fixed-priority algorithm.

Corollary

The RM algorithm is optimal among all fixed-priority algorithms whenever the relative deadlines of all tasks are proportional to their periods.

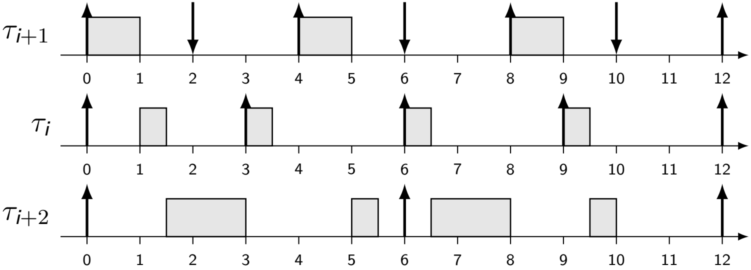
Proof

- We can always transform a feasible static-priority schedule that is not a DM schedule into one that is.
- Suppose $\tau_1, \ldots, \tau_i$ are prioritized not in accordance with DM. Suppose τ_i has a longer relative deadline than τ_{i+1}, but τ_i a higher priority than τ_{i+1}. Then, we can interchange τ_i and τ_{i+1} (switch the priorities) and adjust the schedule accordingly by swapping "pieces" of τ_i with "pieces" of τ_{i+1}.



Proof (cont.)

- After the switch, the priorities of the two tasks are assigned on the DM basis relative to the other tasks.



- By induction, we can correct all such situations and transform the given schedule into DM schedule.
- **Note:** It is always possible to switch the priorities of tasks and hence the time intervals without leading to any missed deadline when tasks are in phase. **Why?**

Outline

Utilization-based RM schedulability test

CZ: Test rozvrhnutelnosti RM na základě zatížení

Theorem (Liu, Layland)

A system of n independent, preemptable periodic tasks with relative deadlines equal to their respective periods can be feasibly scheduled on one processor according to the RM algorithm if its total utilization $U = \sum_{i=1}^{n} u_i$ satisfies:

$$U \leq n(2^{\frac{1}{n}} - 1)$$

$U_{RM}(n) = n(2^{\frac{1}{n}} - 1)$ is the schedulable utilization of the RM algorithm.

Note: This is only a sufficient (not necessary) schedulability test.

U_{RM} as a function of n

n	$U_{RM}(n)$
2	0.828
3	0.779
4	0.756
5	0.743
6	0.734
7	0.728
8	0.724
9	0.720
10	0.717
$\vdots$	$\vdots$
∞	$\ln 2 \approx 0.693$

Proof

See Liu's book.

Other utilization-based tests

Liu's book presents several other utilization-based schedulability tests.

- Some of these tests result in higher schedulable utilizations for certain kinds of task sets.
- Other deal with different task models such as those similar to MPEG decoder.

Outline

Critical instant

The following analysis methods are based on a notion of "critical instant".

Definition (Critical instant)

Critical instant of a task τ_i is a time instant such that:

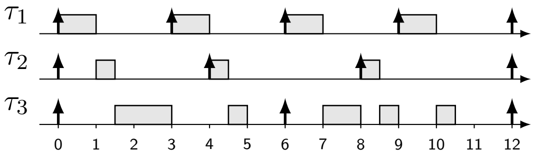
1. the job of τ_i released at this instant has the maximum response time of all jobs in τ_i, if the response time of every job of τ_i is at most D_i, the relative deadline of τ_i and
2. the response time of the job released at this instant is greater than D_i if the response time of some jobs in τ_i exceeds D_i.

Informally, a critical instant of τ_i represents a worst-case scenario from τ_i standpoint.

Critical instant in static-priority systems

Theorem (Liu, Layland)

In a fixed-priority system where every job completes before the next job of the same task is released, a critical instant of any task τ_i occurs when one of its job $J_{i,c}$ is released at the same time with a job of every higher priority task.



Critical instant of τ_3 is 0.

We are not saying that $\tau_1, \ldots, \tau_i$ will all necessarily release jobs at the same time, but if this does happen, we are claiming that the time of release will be a critical instant for τ_i.

Time-demand analysis (TDA)

CZ: Analýza časové poptávky

- Compute the total demand for processor time by a job released at a critical instant of the task and by all the higher-priority tasks.
- Check whether this demand can be met before the deadline of the job.
- TDA can be applied to produce a schedulability test for any static-priority algorithm that ensures that each job of every task completes before the next job of that task is released.
- For some important task models and scheduling algorithms, this schedulability test will be necessary and sufficient.
- Time-demand analysis was proposed by Lehoczky, Sha, and Ding.

Scheduling condition

Definition

The time demand function of the task τ_i, denoted $w_i(t)$, is defined as follows.

$$\boxed{w_i(t) = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil \cdot C_k \quad \text{for } 0 < t \leq T_i.}$$

Note: We are still assuming tasks are indexed by priority.

For any static-priority algorithm $\mathcal{A}$ that ensures that each job of every task completes by the time the next job of that task is released.

Theorem

System $\mathcal{T}$ of periodic, independent, preemptable tasks is schedulable on one processor by algorithm $\mathcal{A}$ if the following holds.

$$\forall i : \exists t : w_i(t) \leq t, \quad 0 < t \leq T_i.$$

Necessity and sufficiency

- Condition $\forall i : \exists t : w_i(t) \leq t, \quad 0 < t \leq T_i$ is necessary for
  - synchronous real periodic task systems and
  - real sporadic task systems.
  - Why?
- For a given i, we don't really have to consider all t in the range $0 < t \leq T_i$. Two ways to avoid this:
  1. Iterate using $t^{(k+1)} := w_i(t^{(k)})$, starting with a suitable $t^{(0)}$ (e.g. $t^{(0)} = C_i$) and stopping when, for some n, $t^{(n)} \geq w_i(t^{(n)})$ or $t^{(n)} > T_i$.
  2. Only consider $t = j \cdot T_k$, where $k = 1, 2, \ldots, i$; $j = 1, 2, \ldots, \lfloor \min(T_i, D_i)/T_k \rfloor$.
     - Explanation is in Liu's book.

Response-time analysis

CZ: Výpočet doby odezvy

- A special (simple) case of Time Demand Analysis
- If we know the critical instant, we can find task's worst-case response time by "simulating" the schedule from the critical instant.
- The found worst-case response time R_i is compared with the corresponding deadline:

$$R_i \leq D_i$$

- The response time can be calculated as follows:

$$R_i = C_i + I_i,$$

  where I_i denotes the interference from higher priority tasks.
- In time interval $[0, R_i)$ the higher priority task τ_j will be executed several times:

$$\text{number of } \tau_j \text{ executions} = \left\lceil \frac{R_i}{T_j} \right\rceil$$

- Total interference from τ_j to τ_i is: $I_{i,j} = \left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Response-time analysis (cont.)

$$R_i = C_i + \sum_{j \in \mathsf{hp}(i)} I_{i,j}$$

$$R_i = C_i + \sum_{j \in \mathsf{hp}(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

Response-time analysis (cont.)

$$R_i = C_i + \sum_{j \in \mathrm{hp}(i)} I_{i,j}$$

$$R_i = C_i + \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

Response-time analysis (cont.)

$$R_i = C_i + \sum_{j \in \mathbf{hp}(i)} I_{i,j}$$

$$R_i = C_i + \sum_{j \in \mathbf{hp}(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

- Can be calculated by the recurrence formula:

$$w_i^{n+1} = C_i + \sum_{j \in \mathbf{hp}(i)} \left\lceil \frac{w_i^n}{T_j} \right\rceil C_j$$

- Sequence $w_i^0, w_i^1, \ldots$ is monotonically non-decreasing. If $w_i^n = w_i^{n+1}$ then it is the solution of the equation. Choice of w_i^0 is important. It should not be greater than R_i. We can start with 0 or C_i.

Response-time analysis example

Example

Task	Period	Execution time
τ_a	7	3
τ_b	12	3
τ_c	20	5

$$R_a = 3$$

$$w_b^0 = 3$$

$$w_b^1 = 3 + \left\lceil \frac{3}{7} \right\rceil 3 = 6$$

$$w_b^2 = 3 + \left\lceil \frac{6}{7} \right\rceil 3 = 6$$

$$R_b = 6$$

Response-time analysis example (cont.)

Example

$$
\begin{aligned}
w_c^0 &= 5 \\
w_c^1 &= 5 + \left\lceil \frac{5}{7} \right\rceil 3 + \left\lceil \frac{5}{12} \right\rceil 3 = 11 \\
w_c^2 &= 5 + \left\lceil \frac{11}{7} \right\rceil 3 + \left\lceil \frac{11}{12} \right\rceil 3 = 14 \\
w_c^3 &= 5 + \left\lceil \frac{14}{7} \right\rceil 3 + \left\lceil \frac{14}{12} \right\rceil 3 = 17 \\
w_c^4 &= 5 + \left\lceil \frac{17}{7} \right\rceil 3 + \left\lceil \frac{17}{12} \right\rceil 3 = 20 \\
w_c^5 &= 5 + \left\lceil \frac{20}{7} \right\rceil 3 + \left\lceil \frac{20}{12} \right\rceil 3 = 20 \\
R_c &= 20
\end{aligned}
$$

Fixed-priority tasks with arbitrary deadlines

- $D_i > T_i \Rightarrow$ More than one job of a task can be ready for execution at a time.
- We assume that jobs are schedules in FIFO order.
- TDA schedulability condition holds only if $d_{i,k} < r_{i,k+1}$

Busy interval

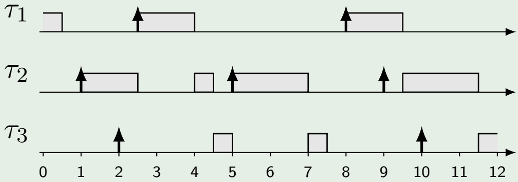
Definition (Level-p_i busy interval)

A level-p_i busy interval$(t_0, t]$ begins at an instant t_0, when

1. all jobs in hp(τ_i) released before the instant have completed and
2. a job in hp(τ_i) is released.

The interval ends at the first instant t after t_0 when all the jobs in hp(t_i) released since t_0 are complete.

Example (Priority $p_i = i$)



Busy interval

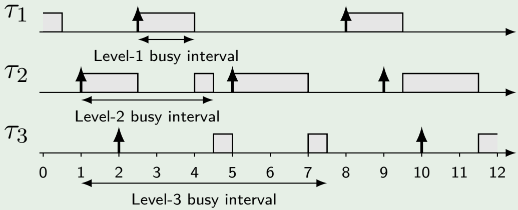
Definition (Level-p_i busy interval)

A level-p_i busy interval $(t_0, t]$ begins at an instant t_0, when

1. all jobs in hp(τ_i) released before the instant have completed and
2. a job in hp(τ_i) is released.

The interval ends at the first instant t after t_0 when all the jobs in hp(t_i) released since t_0 are complete.

Example (Priority $p_i = i$)



Busy interval

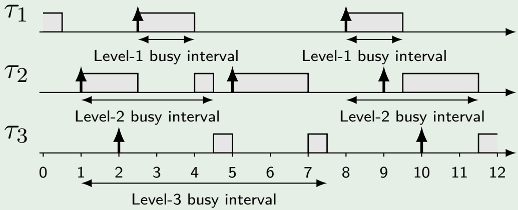
Definition (Level-p_i busy interval)

A level-p_i busy interval$(t_0, t]$ begins at an instant t_0, when

1. all jobs in $\mathrm{hp}(\tau_i)$ released before the instant have completed and
2. a job in $\mathrm{hp}(\tau_i)$ is released.

The interval ends at the first instant t after t_0 when all the jobs in $\mathrm{hp}(t_i)$ released since t_0 are complete.

Example (Priority $p_i = i$)

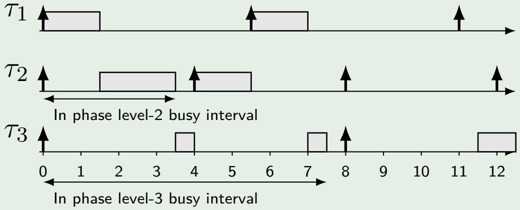


In phase busy interval

Definition

A level-p_i busy interval is in phase if the first jobs of all tasks in $\mathrm{hp}(\tau_i)$ that are executed in this interval have the same release time.

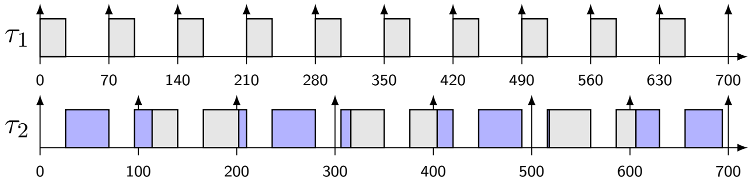
Example (Priority $p_i = i$)



Analysis of tasks with arbitrary deadlines is not as easy

Lehoczky counterexample

- Consider two tasks (T_i, C_i): $\tau_1 = (70, 26)$, $\tau_2 = (100, 62)$.
- RM schedule:



- Seven jobs of τ_2 execute in the first level-2 busy interval.
- Their response times are: 114, 102, **116**, 104, **118**, 106, 94.
- Response time of the first job is not the largest.

Response time calculation

- Test one task at a time from τ_1 to τ_N (see next slides).
- For the purpose of determining whether a task τ_i is schedulable, assume that all the tasks are in phase and the first level-p_i busy interval begins at time 0.
- While testing whether all the jobs in τ_i can meet their deadlines (i.e., whether τ_i is schedulable), consider the subset $\mathrm{hp}(\tau_i)$ of tasks.

Response time calculation (cont.)

1. If the first job of every task in hp(τ_i) completes by the end of the first period of the task, check whether the first job $\tau_{i,1}$ meets its deadline. τ_i is schedulable if $\tau_{i,1}$ completes in time. Otherwise, τ_i is not schedulable.[2]

[2] The same case as if $D_i \leq T_i$

Response time calculation (cont.)

1. If the first job of every task in $\text{hp}(\tau_i)$ completes by the end of the first period of the task, check whether the first job $\tau_{i,1}$ meets its deadline. τ_i is schedulable if $\tau_{i,1}$ completes in time. Otherwise, τ_i is not schedulable.[2]
2. If the first job of some task in $\text{hp}(\tau_i)$ does not complete by the end of the first period of the task, do the following:

[2] The same case as if $D_i \leq T_i$

Response time calculation (cont.)

1. If the first job of every task in $\mathrm{hp}(\tau_i)$ completes by the end of the first period of the task, check whether the first job $\tau_{i,1}$ meets its deadline. τ_i is schedulable if $\tau_{i,1}$ completes in time. Otherwise, τ_i is not schedulable.[2]
2. If the first job of some task in $\mathrm{hp}(\tau_i)$ does not complete by the end of the first period of the task, do the following:
   a. Compute the length of the in phase level-p_i busy interval by solving the equation $t = \sum_{k=1}^{i} \lceil \frac{t}{T_k} \rceil C_k$ iteratively, starting from $t^{(1)} = \sum_{k=1}^{i} C_k$ until $t^{(l+1)} = t^{(l)}$ for some l 1. The solution $t^{(l)}$ is the length of the level-p_i busy interval.

[2] The same case as if $D_i \leq T_i$

Response time calculation (cont.)

1. If the first job of every task in $\mathrm{hp}(\tau_i)$ completes by the end of the first period of the task, check whether the first job $\tau_{i,1}$ meets its deadline. τ_i is schedulable if $\tau_{i,1}$ completes in time. Otherwise, τ_i is not schedulable.[2]
2. If the first job of some task in $\mathrm{hp}(\tau_i)$ does not complete by the end of the first period of the task, do the following:
   a. Compute the length of the in phase level-p_i busy interval by solving the equation $t = \sum_{k=1}^{i} \lceil \frac{t}{T_k} \rceil C_k$ iteratively, starting from $t^{(1)} = \sum_{k=1}^{i} C_k$ until $t^{(l+1)} = t^{(l)}$ for some $l\,1$. The solution $t^{(l)}$ is the length of the level-p_i busy interval.
   b. Compute the maximum response times of all $\lceil t^{(l)} / T_i \rceil$ jobs of τ_i in the in-phase level-p_i busy interval in the manner described next and determine whether they complete in time.

[2] The same case as if $D_i \leq T_i$

Response time calculation (cont.)

1. If the first job of every task in $\text{hp}(\tau_i)$ completes by the end of the first period of the task, check whether the first job $\tau_{i,1}$ meets its deadline. τ_i is schedulable if $\tau_{i,1}$ completes in time. Otherwise, τ_i is not schedulable.[2]
2. If the first job of some task in $\text{hp}(\tau_i)$ does not complete by the end of the first period of the task, do the following:
   a. Compute the length of the in phase level-p_i busy interval by solving the equation $t = \sum_{k=1}^{i} \lceil \frac{t}{T_k} \rceil C_k$ iteratively, starting from $t^{(1)} = \sum_{k=1}^{i} C_k$ until $t^{(l+1)} = t^{(l)}$ for some $l\,1$. The solution $t^{(l)}$ is the length of the level-p_i busy interval.
   b. Compute the maximum response times of all $\lceil t^{(l)} / T_i \rceil$ jobs of τ_i in the in-phase level-p_i busy interval in the manner described next and determine whether they complete in time.
   c. τ_i is schedulable if all these jobs complete in time; otherwise τ_i is not schedulable.

[2] The same case as if $D_i \leq T_i$

Response time calculation of the first job

Step 2b

- Almost the same as in the time demand analysis for $D_i \leq T_i$.
- The time-demand function $w_{i,1}$ is defined as follows:

$$\boxed{w_{i,1}(t) = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil \cdot C_k \quad \text{for } 0 < t \leq w_{i,1}(t)}$$

The only difference is here.

- The maximum possible response time $R_{i,1}$ of job $\tau_{i,1}$ is

$$R_{i,1} = \min\{t | t = w_{i,1}(t)\}$$

- The same iterative computation as in response-time analysis.

Response time calculation for any job in the busy interval

Step 2b

Lemma

The maximum response time $R_{i,j}$ of the j-th job of τ_i in an in-phase level-p_i busy interval is

$$R_{i,j} = \min\left\{ t \middle| t = w_{i,j}\big(t + \underbrace{(j-1)T_i}_{\text{Release time of j-th job}}\big) - (j-1)T_i \right\},$$

where

$$w_{i,j}(t) = jC_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil \cdot C_k \quad \text{for } (j-1)T_i < t \le w_{i,j}(t).$$

- Can be solved by the recurence relation as before.

Response time calculation for any job in the busy interval

Step 2b

Lemma

The maximum response time $R_{i,j}$ of the j-th job of τ_i in an in-phase level-p_i busy interval is

$$R_{i,j} = \min\left\{ t \middle| t = w_{i,j}\big(t + \underbrace{(j-1)\,T_i}_{\text{Release time of j-th job}}\big) - (j-1)\,T_i \right\},$$

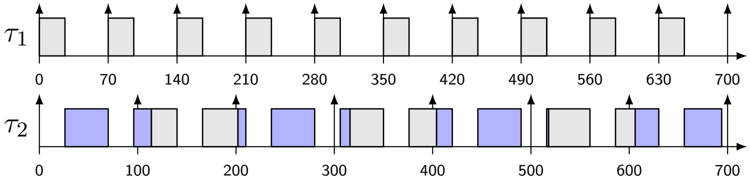
where

$$w_{i,j}(t) = jC_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil \cdot C_k \quad \text{for } (j-1)\,T_i < t \leq w_{i,j}(t).$$

- Can be solved by the recurence relation as before.

Example

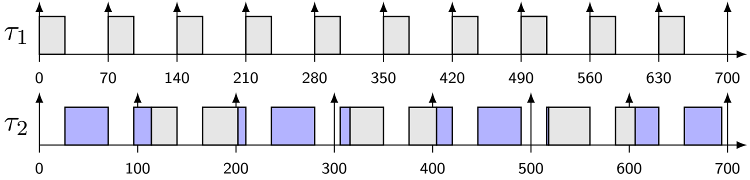
Let us apply the previous Lemma to the Lehoczky example:
$\tau_1 = (70, 26)$, $\tau_2 = (100, 62)$.



- 1) Does not apply because $R_{2,1} > T_2$.
- 2a) The length of level-2 busy interval is 694.
- 2b) Compute response times $R_{2,j}$ for $1 \leq j \leq \lceil 695/100 \rceil = 7$.

Example – calculation of $R_{2,1}$

$\tau_1 = (70, 26)$, $\tau_2 = (100, 62)$.



$R_{2,1}$ = minimal t satisfying:

$$t = w_{2,1}(t) = C_2 + \sum_{k=1}^{2-1} \left\lceil \frac{t}{T_k} \right\rceil \cdot C_k$$

$$= 62 + \lceil t/70 \rceil \cdot 26$$

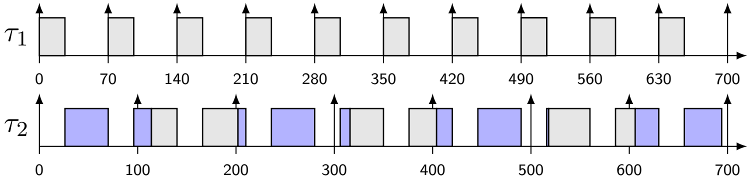
Try substitute 114 for t:

$$114 = 62 + \lceil 114/70 \rceil \cdot 26 =$$
$$= 62 + 2 \cdot 26 =$$
$$= 114 \text{ OK!}$$

What if we don't know what to substitute?

Example – calculation of $R_{2,2}$

$\tau_1 = (70, 26)$, $\tau_2 = (100, 62)$.



$R_{2,2}$ = minimal t satisfying:

$$t = w_{2,2}(t + T_2) - T_2 =$$

$$= 2C_2 + \sum_{k=1}^{2-1} \left\lceil \frac{t + 100}{T_k} \right\rceil \cdot C_k - 100 =$$

$$= 124 + \lceil (t + 100)/70 \rceil \cdot 26 - 100$$

Try substitute 102 for t:

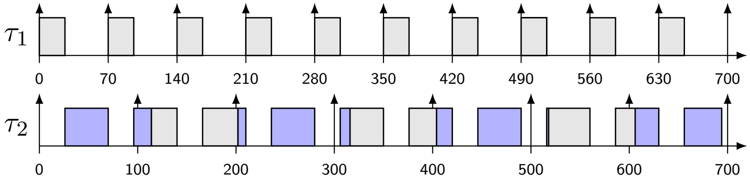
$$102 = 124 + \lceil 202/70 \rceil \cdot 26 - 100 =$$

$$= 124 + 3 \cdot 26 - 100 =$$

$$= 102 \text{ OK!}$$

Example – calculation of $R_{2,3}$

$\tau_1 = (70, 26)$, $\tau_2 = (100, 62)$.



$R_{2,3}$ = minimal t satisfying:

$$t = w_{2,3}(t + 2T_2) - 2T_2 =$$

$$= 3C_2 + \sum_{k=1}^{2-1} \left\lceil \frac{t + 200}{T_k} \right\rceil \cdot C_k - 200 =$$

$$= 186 + \lceil (t + 200)/70 \rceil \cdot 26 - 200$$

Try substitute 116 for t:

$$116 = 186 + \lceil 316/70 \rceil \cdot 26 - 200 =$$

$$= 186 + 5 \cdot 26 - 200 =$$

$$= 116 \text{ OK!}$$

Outline

Summary

- RM and DM are optimal among fixed priority scheduling algorithms.
- Utilization based test is simple but only a sufficient condition.
- Response time analysis is both sufficient and necessary.
- Generic time demand analysis can handle many cases not covered by the response time analysis.